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SEASAT ALTIMETRY ADJUSTMENT MODEL INCLUDING TIDAL AND OTHER SEA SURFACE EFFECTS

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This study is a part of the on-going effort aimed at improved determinations of the earth's gravity field through the adjustment of satellite altimeter data in the short-arc mode. Until recently the key role in such adjustments has been played by GEOS-3 altimeter data. However, SEASAT altimetry is envisioned as providing a more accurate means for addressing this task.

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In view of the improved quality of altimeter data and of the corresponding more stringent requirements for the data reduction, several improvements in the existing altimetry model have been designed and are described herein. For example, the criteria have been established specifying the maximum and the minimum allowable lengths of SEASAT arcs. An important improvement in the economy has been achieved through a reduction in the number of spherical-harmonic potential coefficients entering the orbital integrator, without a noticeable compromise in the excellent quality of the SEASAT observational system.

In a parallel development, the satellite altimetry model has been improved by allowing for the inclusion of certain sea surface effects. The most important in this respect are the tidal effects (long-period, diurnal and semidiurnal), which are now subject to adjustment within the overall adjustment of SEASAT altimetry. Other effects can be included in the form of corrections to altimeter measurements. This development continues along several lines, such as adjusting a greater number of tidal constituents, or adjusting the tidal phases in addition to their amplitudes.

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1. INTRODUCTION

The on-going effort whose various facets have been described e.g. in [Blaha, 1975, 1977, 1979, 1980] is concerned with the adjustment of satellite altimeter data in two steps, the first performed in terms of a truncated set of spherical-harmonic (S.H.) potential coefficients and the second performed in terms of point-mass (P.M.) magnitudes as parameters. The emphasis has been shifted recently toward SEASAT altimeter data as the main source of observed quantities in the first adjustment; gravity anomalies and other sources of geopotential information have been included via the weighted S.H. coefficients. At this stage (first adjustment), six weighted state vector (s.v.) parameters per orbital arc are also included in the simultaneous least-squares process. The first adjustment is global in character. One of its most important products is a revised set of S.H. coefficients which may be of interest in itself, and which is especially useful in predicting geoid undulations, gravity anomalies and other quantities related to the disturbing potential (such as deflections of the vertical or gravity gradients) on the global scale.

The data for the second adjustment consist mainly of the residuals from the first adjustment, although other quantities (gravity anomalies, deflections of the vertical, etc.) can enter this phase independently. In this process a more detailed, but regional, geoid is derived. Predictions of the other quantities just mentioned can also be made in the region of interest. A given set of point masses has a chosen distribution which, as a rule, is uniform and is characterized by the 1.6:1 depth-side ratio.

Denser sets of point masses can be superimposed on the basic set, leading to an even more detailed description of the gravity field in specific sub-regions. This can be related to the mean values of geoid undulations derived, for blocks of a certain size, from satellite altimetry. In some areas of pronounced and varied geoidal relief, such as in the Puerto Rico trench area, the differences between the mean and the actual undulations could become large and, thus, smaller blocks might be chosen to describe the geoid. From geoid undulations (or their means) one could derive other quantities related to the disturbing potential, upon using the appropriate cross-covariance functions. The present approach with point masses circumvents, by construction, the need for these functions.

One could contemplate using the P.M. parameters directly in conjunction with the ellipsoidal reference field, without the intermediary of the first adjustment. However, in addition to losing the possibility of revising the values of the S.H. coefficients and the s.v. parameters as well as of obtaining global predictions of the desired geophysical quantities, one would also introduce modeling errors due to the spherical approximation in the P.M. model; on the other hand, the "anomalous" quantities relative to the S.H. field are much smaller (an order of magnitude) than those relative to the reference field, which renders the spherical approximation inconsequential. It can also be mentioned that certain geoidal detail is already described by the actual altimeter observations in conjunction with the adjusted s.v. parameters. However, this detail is provided only along one dimension (i.e., along the satellite pass). Other means, such as the P.M. parameters, are needed to describe, to within a desired resolution, the geoidal detail as well as other geophysical quantities in two dimensions. One final product obtained with the P.M. adjustment superimposed on the S.H. adjustment is a set of contour maps based on predicted values for geoid undulations, gravity anomalies, etc. A typical example is a geoidal map in a region containing point masses. This region exhibits detailed geoidal features, while far from it the geoid described by the potential coefficients alone is very smooth; the transition from one region to the other is gradual.

In recent adjustments of GEOS-3 and SEASAT altimeter data, the S.H. model has consisted of a (14,14) set of potential coefficients and the P.M. model has consisted of some 150-200 point masses distributed essentially in an equilateral grid covering a region of interest such as the North Atlantic. In agreement with an earlier statement, the depth of point masses has been stipulated to be approximately 1.6 times their horizontal separation and only their magnitudes have been subjected to an adjustment. It is considered that good predicted values to within the desired resolution for both N (geoid undulations) and Δg (gravity anomalies) can be obtained if the shortest half-wavelength to be represented by the P.M. adjustment corresponds to two point masses in each dimension of the P.M. grid. From this point of view, if the point masses form a $2^0 \times 2^0$ grid, the geoidal resolution is regarded with confidence to within 4^0 features. Details on the real data reductions recently performed in this fashion using GEOS-3 and SEASAT altimetry can be found in [Blaha and Hadgigeorge, 1979] and [Hadgigeorge et al., 1980], respectively.

In this study, the adjustment capabilities are extended with specific considerations given to SEASAT altimetry and its precision. Chapter 2 describes several improvements in the existing adjustment algorithm. Criteria are developed with regard to the maximum and minimum allowable length of satellite arcs, and to the selection of a suitable observational density on an arc. A feature leading to significant reductions in the computer run-time requirements is developed, characterized by a reduction in the number of constants entering the orbital integrator in the exercise of the short-arc algorithm. Other features are discussed, such as the possibility of reducing the number of s.v. parameters from six to four per arc, or the option to artificially lower the input sigma (square root of the variance) of the s.v. parameters in order to prevent them from absorbing the tidal effects, in case the latter are not included in the first adjustment.

The tidal and other sea surface effects are subsequently discussed in Chapter 3. Due to a higher precision of SEASAT altimeter and the satellite ephemeris as compared to the GEOS-3 system, modeling errors caused by certain sea level changes can no longer be ignored. The most important changes are those caused by the tide-generating forces of the moon and the sun. The long-period, diurnal and semidiurnal tidal effects are initially discussed for a theoretical model (equilibrium tide). Based on the relative importance of these effects the decision is made as to which of them are to be described herein and which will be described in the next report. The chapter culminates with a practical tidal adjustment and its incorporation into the SEASAT adjustment model. A possible inclusion of a few other sea surface effects is also considered.

2. IMPROVEMENTS AND FURTHER DEVELOPMENT OF SATELLITE ALTIMETRY ADJUSTMENT ALGORITHM

When addressing the problem of extending the altimetry adjustment algorithm from GEOS-3 to SEASAT, one should keep in mind a better global coverage and a higher resolution power of the latter (0.1 to 0.2 m altimeter sigma for SEASAT as compared to about 1 m for GEOS-3). Furthermore, the input state vector (s.v.) parameters for each satellite arc, obtained from the precise ephemeris, are characterized by approximately 2 m sigma in position as compared with 10 to 20 m sigma associated with the broadcast ephemeris used previously.

Maximum arc-length criterion. In considering the above differences, a new criterion for the length of satellite arcs has been established. This criterion has resulted from computer simulations and has been confirmed in principle during the real data reductions. Since the short-arc concept is the cornerstone of all the altimeter reductions performed in the first step (S.H. adjustment), it will be now briefly reviewed.

In the short-arc adjustment algorithm, developed for satellite altimetry by Brown [1973], the S.H. coefficients entering the orbital integrator have the role of (fixed) constants. Since these constants have errors associated with them which cannot be corrected or modified by the adjustment, one is faced with a model deficiency which may or may not be detrimental to the altimeter adjustment. If the arc is very short, only a few low-degree and order coefficients will have any bearing on its shape. The altimeter misclosures (constant terms) in the observation equations will not be affected to any significant degree because such coefficients are known to a high degree of accuracy. However, as the arc becomes longer the misclosure distortions tend to get larger, increasing from zero at the mid-arc epoch to a maximum at the extremities. They are of the same sign and, when plotted, give rise to a curve resembling a parabola and indicating an orbital curvature error. This is imputable to errors in higher-degree and order coefficients including the truncation of the set. Therefore, the errors in the initial S.H. coefficients propagate into the misclosures of observation equations as a function of the errors' magnitudes and of the arc's length.

The above qualitative statement was at the root of the study performed in view of SEASAT altimetry in Chapter 3 of [Blaha, 1979] and summarized in [Blaha, 1979']. Various computer simulations have indicated that the systematic errors resulting from the truncation of the coefficients beyond the degree and order (6,6) can exceed 1-2 m in the radial component if the arc's duration is about 8 minutes. However, the purpose of the above two references has not been merely to evaluate the misclosure distortions, but to find under what circumstances these distortions can be accommodated by slight adjustments to the state vector parameters. In considering SEASAT observational sigma of 0.1 - 0.2 m, it has been concluded that the distortions can be rendered relatively unimportant -- and thus of little or no consequence for the subsequent P.M. adjustment -- if the arcs' duration does not exceed seven minutes. This arc length was subsequently accepted as the criterion for SEASAT data reductions.

The first adjustment ultimately producing the altimeter residuals as well as corrections to the weighted parameters (S.H. potential coefficients and six state vector components per short-arc) has yielded, as a by-product of an initial stage, the misclosures in the observation equations. These misclosures reflect on the errors in altimeter measurements (the "noise" of the system), in the a-priori state vector parameters, and in the S.H. potential coefficients and the reference field parameters. But, most of all, they reflect on the geoidal detail ignored by the adjustment. Such detail depends on the truncation of the S.H. model and can be represented by a variance, called here the "theoretical variance", obtained as a sum of degree variances for geoid undulations where the summation extends over all the neglected degrees. Thus, in the model truncated at degree and order (14,14) the summation extends over the degrees 15 through perhaps 1000 (beyond this degree the contribution is negligible for all practical purposes). The degree variances are in turn computed from the covariance function.

Upon computing a root mean square (RMS) of the misclosures in the (14,14) model, it has been observed (see Appendix 1) that this value is appreciably lower than the "theoretical sigma", i.e., the square root of the theoretical variance. Yet, this RMS contains contributions from the other sources, i.e., from the ephemeris, the S.H. potential coefficients together with the reference field parameters, and SEASAT altimetry. The low RMS value serves as an indicator of excellent quality inherent in each of the three sources above and, in addition, as an indicator of sufficient accuracy in the short-arc algorithm applied in conjunction with the seven-minute arc criterion. This is supported by the fact that the mean value of the misclosures is nearly zero (-0.1 m). The RMS value actually suggests that the theoretical formula for the covariance function may be too conservative, at least insofar as the geoid undulations for relatively low degree and order truncations are concerned.

Minimum arc-length criterion. Parallel with the analysis just described, an effort has been undertaken to eliminate the arcs which would be too short for a meaningful adjustment of SEASAT altimetry. An arc which would be essentially isolated (too short to intersect with other arcs) could lead to the situation where an important geoidal detail would be absorbed, partly or entirely, by the corrections to the state vector parameters. Arcs' intersections with other arcs at crossover points ensure a cantelever effect preventing any individual set of state vector parameters from masking the geoidal detail. The degree of this prevention depends on the number of intersections and thus on the arc's length, in the sense the longer, the better. However, the arcs of merely 3^0 in angular length will already be satisfactory as is explained next.

In order to prevent a short arc from absorbing, over its length, a constant raise in the geoid, at least one intersection is needed. However, the arc can still absorb a geoidal tilt around the axis through the crossover point perpendicular to the orbital plane, as well as a curvature change. One additional intersection will then prevent the tilt change from taking place and another one will do the same for the curvature change. It thus follows that the arc's length should allow for at least three intersections with other arcs. In this way the residuals entering the more detailed, P.M. adjustment will not exhibit unrealistically small magnitudes caused by the arcs' absorption of one or more of the above effects. If the arcs are at least 3^0 in length, there should be in general at least three intersections, except perhaps in some isolated cases where a satellite pass had been previously eliminated at the pre-processing and screening stages. The elimination of arcs under 3^{0} in angular length or, equivalently, under 50.4 seconds in duration will not deplete the amount of usable data since such arcs have been found to amount to no more than 10-20% of all arcs and, more importantly, to contain merely 1-2% of observations. By comparison, about one-half of all arcs are 25^{0} in angular length stemming from the seven-minute criterion.

Observational density on an arc. Another problem addressed in this study is related to the density of SEASAT observations along one dimension (i.e., along the pass) in contrast to the density of the ground tracks intersecting essentially in a $1^{0} \times 1^{0}$ grid formed by ascending and descending passes. Adopting only the crossover observations (generated by an interpolation) would be detrimental to the state-vector adjustment in that only a few degrees of freedom would remain, that the arcs under 7^{0} or 8^{0} in angular length would have to be eliminated, a detailed plotting of a point-to-point geoid along the passes would be inhibited, etc. On the other hand, the other extreme of utilizing all the altimeter measurements would be economically prohibitive from the computer run-time standpoint. Furthermore, little would be gained from a configuration where the density of observations along one dimension is over 30 times as high as the density along the other dimension. A conclusion has been reached in the form of a compromise by accepting every 8th point on each arc for the adjustment (other options are also available). In this way, the separation between measurements along tracks is $\frac{1}{4}^{0}$ while the separation across tracks is 1^{0}. This also allows for a reasonable amount of redundancy in case one attempted to construct a $2^{0} \times 2^{0}$, or even $1^{0} \times 1^{0}$, geoid.

Construction of an "observed" geoid along satellite passes. An "observed" geoid, which may serve in various geophysical, geodetic and oceanographic analyses, has been designed to be plotted along each satellite arc in much the same way as the global geoid from the first (S.H.) adjustment. The quotes are used in order to distinguish this geoid from the more directly observed geoid as it would appear before any kind of adjustment. The latter geoid is given essentially as the initial radial distance (from the geocenter) to the satellite minus the radial distance to the ellipsoid minus the altimeter measurement; a small correction due to the earth's center, the altimeter foot-point, and the satellite not being in a straight line has been discussed in [Blaha, 1977]. The observed geoid, although known as a by-product of the current misclosure computation, is not envisioned to be used for plotting. Instead, the "observed" geoid is recommended for this purpose; it differs from the observed geoid only in that the adjusted rather than the initial radial distance to the satellite is adopted in its computation. It still contains the same high-frequency information, but is improved overall through the corrections to the state vector parameters as determined in the global adjustment. Accordingly, errors in this geoid stemming from the errors in the ephemeris are expected to be reduced or eliminated. The short-arc adjustment algorithm thus proves useful in producing a detailed and reliable geoidal profiles along the satellite passes, consistent with the high quality of SEASAT altimeter observations.

Reduction in run-time requirements. In considering that the computer time consumed by the computations carried out by the orbital integrator accounts for a major portion of the total computer time used in the global adjustment of satellite altimetry, the possibility has been sought to reduce the run-time requirements without compromising the high quality of SEASAT altimetry. The main function of the orbital integrator is to compute the satellite positions at given instants which correspond to SEASAT events (altimeter observations). Needed for this task are the s.v. parameters associated normally with the mid-arc epoch and the S.H. potential coefficients entering the integrator in the role of constants used in the computation of all the other points on the arc. Previously these constants consisted of the same S.H. potential coefficients which entered into the adjustment of the global geoid (weighted terrestrial parameters). The set of these coefficients has been typically truncated at the degree and order (14,14).

However, it has been considered that for the orbital integrator this set could be further truncated without introducing inadmissible errors. The main feature of the short-arc algorithm which makes such economies feasible is the property of the arcs being "short", in the sense that the longest arc in a SEASAT adjustment has been stipulated not to exceed 7 minutes in duration. If the epoch is at mid-arc, the longest time interval which the integrator would have to span is then 3.5 minutes. This interval is sufficiently short to allow for a reduction in the number of coefficients used by the integrator, while at the same time maintaining the altimeter errors thus introduced at levels substantially smaller than SEASAT altimeter noise. The problem at hand is thus the following: How much can the coefficient set be truncated for the use in the orbital integrator without compromising the excellent quality of SEASAT altimeter measurements?

This problem has been addressed by forming the observation equations at two levels; the first level represents the standard approach in which the "full" set, here the (14,14) set, is used in both the geoid computation and the orbital integrator, and the second level represents the new approach with a "reduced" set entering the orbital integrator, everything else being the same (SEASAT observations, s.v. parameters, S.H. parameters used in the geoid computation, etc.). The tests conducted during this research have begun with a (10,10) reduced set using randomly selected SEASAT passes filed on various AFGL magnetic tapes and considered to be fairly representative of the world's oceans and of the gravity field affecting the satellite. A detailed description follows.

The most extensive of these tests, in which all of the misclosures and their statistics have been printed, involves 38 SEASAT arcs registered on the tape no. CS1700. Almost two-thirds of the arcs approach the maximum allowable length of 6 or 7 minutes in duration while the rest of the arcs are shorter, quite typically about one-half of this length (about 3 minutes in duration). The epoch is considered to be at the mid-arc, especially where the longest arcs are concerned. The difference between the two parallel sets of observation equations are alternately called "errors" because they have the nature of errors in the radial direction, provided the results obtained with the full set are taken as an errorless standard. Such errors will clearly affect the quality of SEASAT altimetry and will have to be carefully scrutinized. The sum of these errors tends toward zero as the number of arcs increases. As will be explained later, these errors have random characteristics on the whole (although not on an individual arc) and will be regarded as having a

Gaussian (normal) distribution with the zero mean and a certain variance. Their sigma (the square root of the variance) will be compared with the sigma of the SEASAT altimeter noise in order to determine the acceptability of a given reduced set of coefficients.

In about 85% of the arcs the differences in misclosures (i.e., the errors) are of the same sign; they start with zero values at the epoch and at some distance away from it, and increase geometrically toward the extremities, giving rise to a curve resembling a parabola. The average error for the arc is, except perhaps for the sign, also the average absolute error. The latter does not have to be computed from the printed misclosures for the arc but is known immediately as the difference between the two printed average values for the misclosures (the average misclosure in the reduced set minus the average misclosure in the full set for the same arc). In the remaining 15% of the arcs the situation is complicated by the fact that the errors change sign (once); they are of one sign and of small magnitude at one extremity, reach the zero value at a relatively small distance from it and end up with the opposite sign and a much larger magnitude at the other extremity of the arc. Thus the difference between the average misclosures is somewhat smaller than the above average absolute error. However, this error can be computed individually in several instances and the results can be then used to deduce an approximate factor which will carry out the conversion from the ideal case where the misclosures on all arcs are considered without the sign change to the realistic case where the sign changes in 15% of the arcs. For the 38 arcs examined, this factor, to be applied in the final result computed "blindly" from the average misclosures, was found to be 1.07. This procedure makes it possible to examine dozens of arcs in a matter of hours instead of days, considering that many thousands of individual misclosures may be involved.

When studying the 38 selected arcs it became apparent that the closeness of the epoch to the mid-arc is very important. For a six-minute arc whose epoch is, for example, 4.5 min. from one extremity and only 1.5 min. from the other, it may happen that the extremity farther from the epoch exhibits exceedingly large errors, perhaps over 40 cm, while the errors toward the other extremity may be close to zero. On the other hand, if the epoch is at mid-arc the errors at both extremities of the same pass may be contained within 10 cm; it has been observed that significant deterioration in conjunction with the (10,10) reduced set starts taking place at points about 3.5 to 4 minutes from the epoch.

The reduced set resulted in the errors which in about 1/3 of the cases reached over 20 cm at the extremities; in a few isolated cases these errors surpassed 30 cm. The overall average absolute error was found to be 4.4 cm. It was computed using the procedure described earlier, in which the results from 85% of the arcs are used directly and the remaining 15% of the arcs with changing signs of the errors are treated in detail in order to produce comparable results. About 73% of the errors were found to be contained within 5.5 cm (the misclosures in the two parallel computer runs are rounded to the nearest cm, and this outcome was reached by counting the incidence of errors having the magnitude of 6 cm or more according to the computer printouts). The errors have a random character insofar as the geoidal determination is concerned; the ascending and descending arcs intersect at numerous locations and where one arc has a "plus" error another may have a "minus" error or no error at all if the intersection takes place near the epoch. Although some of the relatively long arcs may exhibit larger errors than the shorter arcs, they also intersect at a larger number of locations which tends to even things out in the geoidal adjustment.

For the sake of interest, the altimeter observations on the 38 arcs were subjected to a least-squares adjustment, together with the GEM 10 (weighted) S.H. potential coefficients and the s.v. parameters. Although the input sigma in the radial component of the s.v. parameters was 1.6 m, the average absolute differences in the corresponding corrections was merely 3 cm. Similarly, the corrections to the three velocity components differed by a small fraction of the input sigmas (these were on the order of 5 cm per second). Although the radial velocity component was weighted somewhat more loosely than the two horizontal velocity components, the corrections it exhibited were often the smallest of the three. In these comparisons, the geoid (computed with the 14,14 S.H. model) showed differences ranging between 1 and 3 cm, under 1.5 cm on the average. When the residuals were examined, the largest differences between the two adjustments were found to be typically about 3 cm; in a few cases (about 15%) they were in the vicinity of 10 cm. Since the number of passes was too low to allow for arcs' intersections, one may expect that in a global adjustment more of the errors would be accommodated by the changes in the s.v. velocity components and thus by the changes in the arc's curvature. The differences in the S.H. geoid and in the residuals would then be even smaller than those just mentioned. This means that the final geoid, obtained from the S.H. adjustment and from the residuals produced by this adjustment and subjected to a separate modeling, would contain errors (attributable to the 10,10 reduced set used by the orbital integrator) which would reach a few cm at the most and would often be close to zero.

As stated earlier, the errors due to a reduced set are considered random and are evaluated against the background of the altimeter noise. The degradation of the SEASAT altimetry can be assessed from the "combined" sigma,

$$\sigma_{\text{combined}} = (\sigma^2_{\text{altimetry}} + \sigma^2_{\text{reduced set}})^{\frac{1}{2}} .$$

Although the sigma of altimeter noise is given between 10 and 20 cm, the evaluation will proceed under a stricter condition of 10 cm to be used in conjunction with the "combined" sigma. On the other hand, assuming the Gaussian distribution associated with the above errors one can estimate its sigma as the average absolute error (here 4.4 cm) divided by 0.80, namely

$$\sigma_{\text{reduced set}} \simeq 5.5 \text{ cm} ,$$

which turns out to be the magnitude within which 73% of the errors were found by a simple count. In theory, the percentage of such errors within $\pm\sigma$ would be 68% which agrees quite well with the present count using an approximate method and a relatively small sample. In other words, the assumption of Gaussian distribution is reasonably well supported also from this point of view. Upon utilizing the two sigmas just presented, it follows that

$$\sigma_{\text{combined}} \simeq 11.4 \text{ cm} = 1.14\, \sigma_{\text{altimetry}} .$$

This result has been obtained when considering a 3.5 minute time interval from the epoch to an extremity of an arc. In particular, the epoch for the arcs of maximum allowable length (up to 7 minutes in duration) has been considered to be at mid-arc.

According to the above, the sigma of SEASAT observational noise has been degraded by 14% (if $\sigma_{altimetry}$ = 15 cm were taken, this number would be only 6.5%). Such a degradation is not serious and is certainly more than offset by significant computer savings which can be materialized by using only 121 out of 225 coefficients in the orbital integrator. These coefficients serve in the computation of satellite positions on each arc which is the most time-consuming procedure in a global adjustment of satellite altimetry. That similar savings are significant becomes clear if one bears in mind that a global adjustment of SEASAT altimetry involves some 10,000 arcs.

For comparison purposes, 33 passes have been selected at random from another AGFL tape (tape no. CS700). The altimeter observations were not adjusted, only the statistics for the misclosures (average, rms, average absolute value) were printed for each arc. In agreement with an earlier statement, the average absolute value for the errors was first computed "blindly", by comparing the average misclosures obtained with the full and (10,10) reduced sets, and the result was then multiplied by a factor 1.07. The average absolute error thus obtained is somewhat lower than 4.4 cm found earlier, but the earlier result is considered more reliable.

Using the same method and the same 33 passes, a reduced set (8,8) was examined. The average absolute error for these 33 arcs shows an increase by a factor 1.81 when compared with the above (10,10) case. This would ultimately imply a 40% degradation (as opposed to a 14% degradation). This approach could still be acceptable in most practical cases, but should be viewed with caution and special attention should be paid to having the epoch always almost exactly at mid-arc.

Finally, the same test was carried out using a (6,6) reduced set. This time the increase in the average absolute error resulted in a factor 2.93 and the corresponding degradation amounted to almost 90% (the sigma due to the reduced set was over 50% larger than the sigma for the altimeter noise used above).

These experiments have indicated that a (6,6) reduced set should not be used. Perhaps the best outcome, allowing for significant economies, can be achieved with a (10,10) reduced set. If computer savings are of paramount importance, an (8,8) reduced set could also be used.

Unsuitability of reducing the number of state vector parameters. An analysis performed in order to see whether substantial computer savings could be realized by adjusting four rather than six s.v. parameters per arc in a short-arc mode of satellite altimetry is described in detail in Appendix 2. It has been considered that if a satellite orbit is circular or nearly so, if the epoch is at mid-arc and if the arcs are sufficiently short, the directions of altimeter measurements coincide approximately with the "o" direction of the state vector orthogonal triad ("i" indicates in-track, "c" crosstrack, and "o" is orthogonal to the other two, completing a right-handed triad). In that case, adjusting the state vector's position components in "i" and "c" directions would have little bearing on the geoidal adjustment since the altimeter measurements would be almost completely insensitive to differential changes along these two directions. For this reason, also the a-posteriori sigmas in these directions would be hardly improved at all compared to the a-priori values (this has been confirmed during computer simulations). Accordingly, the two position components corresponding to these two directions could be held fixed without introducing any undue strain in the adjustment or significantly altering the geoidal parameters. The advantage of such an approach consists in that these two parameters could be effectively eliminated from the adjustment so that only four state vector parameters -- instead of the original six parameters -- would be adjusted for each arc.

The above possibility seemed attractive especially when considering the thousands of short arcs of satellite altimetry that have become available in recent years. During the adjustment process an inversion of a 6x6 matrix $(\ddot{N}_i + \ddot{P}_i)$ has been made in conjunction with each arc (see e.g. the formulas in Chapter 2 of [Blaha, 1975]), corresponding to the presence of six state vector parameters per arc. The number of algebraic operations (namely, scalar multiplications which consume by far the most computer time) needed in inverting this matrix is (proportionate to) $6^3=216$, while the corresponding number of operations in inverting a 4x4 matrix would be only $4^3=64$. Thus, if nothing except one inversion should be performed for each arc, only about 30% of computer time would be needed and impressive savings of 70% would be realized. Savings in terms of computer storage need not be considered since in the short arc algorithm, the space used by the first arc is reused by every consecutive arc.

The cause for concern in pursuing this avenue has been the fact that if the orbit is not quite circular, the directions of individual measurements along a short arc may depart significantly from the direction "o" of the state vector triad even for very short arcs. The larger the orbit's eccentricity, the greater this departure and the ensuing deformation in the adjustment results. This indicates that the above suggested procedure is a trade-off between the computer efficiency and the rigor of the solution. It is also clear that final computer savings are only a fraction of those estimated by merely adding the total number of scalar multiplications in matrix manipulations; a substantial part of computer run-time is absorbed by the formation of partial derivatives and constant terms in observation equations, not to mention the input-output and other computer operations. It has thus become apparent that the suggested procedure could be useful only if the satellite orbit were nearly circular and if the computer savings as obtained by adding the above scalar multiplications amounted to at least 20% or 30%.

However, during an analysis of the adjustment process it has transpired that the inversions of the earlier mentioned 6x6 matrices associated with the six adjustable state vector parameters account for only a miniscule part of the total number of operations performed in a realistic adjustment. Although certain matrices in the suggested procedure would have one dimension reduced from 6 to 4 (this would lead to computer savings in several matrix multiplications), the overall savings would not be impressive. In particular, if only 100 measurements were recorded along each arc and the total number of arcs were 100, the savings attributed to matrix manipulations would amount to about 6.2%; if the number of arcs increased to 1000 or more, these savings would converge to about 6.4%. If 1000 measurements were recorded (and utilized) along each arc and 100 or more arcs participated in the adjustment, the savings would amount to about 2.2%. Thus even in the best practical case the total computer savings (including the formation of partial derivatives, etc.) could hardly reach 1% and most often would be only a fraction of this number. It is concluded that the efficiency of the existing adjustment algorithm cannot be noticeably improved by this or a similar procedure; moreover, such a procedure could impair the rigor of the solution. The existing algorithm has thus proved to be advantageous and should be retained.

Option to lower the state vector sigmas. Initially, the objective of the first, global adjustment has been to simultaneously solve for the s.v. parameters and a (truncated) set of the S.H. parameters. The objective of the second, regional adjustment built on the residuals from the first adjustment has been to solve for the point-mass (P.M.) parameters and, at a later stage, for the parameters associated with the chosen long-period, diurnal and semidiurnal tides. If one chooses to pursue this approach and to eliminate tidal considerations from the first adjustment in any form (i.e., the tidal effects are not subject to adjustment nor accounted for by suitable corrections), one is faced with the problem of the s.v. adjustment accommodating some, or most, of the tidal effects. This follows from the realization that the tidal signal can resemble a systematic orbital error over local regions in that the wavelengths can be comparable. The tidal effects vary, both in sign and magnitude, from one area to the next and from one time epoch to another. In the present context of SEASAT arcs limited to 7 minutes in duration (or 25° in arc), a tidal effect could manifest itself essentially as a constant rise, or fall, over an arc's length. If the radial components of the state vectors were nearly perfect and were accordingly held fixed or heavily weighted in the first adjustment, the time variations in the sea surface would indeed be reflected in the residuals. The desired tidal constituents could then be modeled and solved for in the second adjustment.

In reality, however, the weight associated with the radial component is usually too weak to ensure such an outcome. The sigma in the radial component given for the SEASAT ephemeris is 1.6 m and roughly compares with the tidal range in open ocean. Thus, if not modeled, a substantial part of the tidal effect could be absorbed by the satellite arc. This problem may be circumvented by an artificial decrease of the state vector's positional sigmas. Although only the radial component is of importance here, a change in the other two positional sigmas (associated with the in-track and crosstrack components) has practically no bearing on the residuals, and it results in a more advantageous algorithm. The procedure to artificially lower the input sigmas of the s.v. parameters during a latter stage of an adjustment is described in detail in Appendix 3.

It has been realized that in this way, the s.v. parameters would lose at least some of their ability to compensate for the actual orbital errors present in the ephemeris. Such an artificiality could be deemed acceptable only by virtue of the good quality of the precise ephemeris as confirmed through data reductions of SEASAT altimetry. Since the positional corrections to the state vectors are reduced or even suppressed, it is important that the original values be unbiased. In this way the errors propagating from the arcs into the residuals behave in a random fashion from arc to arc.

With regard to the variance-covariance propagation, the artificial reduction of the positional sigmas is clearly unjustified. Therefore, one could consider increasing the weights only in the process of computing the corrected s.v. parameters themselves and subsequently computing the residuals; the variance-covariance estimates for the state vectors should be presented in terms of the original, more realistic values. The change in weights should not affect any of the $\dot{X}$ parameters (corrections to the S.H. potential coefficients), their variance-covariance matrix, etc., either, because these values represent an outcome of a least-squares process in which realistic weighting has been applied for all the parameters. Accordingly, this change should take place only in the final stage of the first adjustment, when the solution for $\ddot{X}_i$ (corrections to the six s.v. parameters on the i-th arc) is "unfolded" from the overall adjustment of $\dot{X}$.

An important outcome of these considerations is that the residuals from the first adjustment carried out with artificially lowered sigmas of s.v. parameters would contain, in addition to the essentially unmodeled orbital errors (assumed random from arc to arc), also the unaltered tidal effects and the geoidal features. The second adjustment would then proceed to model the tidal effects by appropriate tidal parameters, and the geoidal detail by the P.M. parameters. The various effects of random character including the altimeter noise, unmodeled geoidal detail and the above unmodeled orbital errors, as well as unmodeled sea-surface effects, would manifest themselves in second-generation residuals.

However, a subsequent development has indicated that the need for the above approach will be greatly diminished if a tidal adjustment is implemented in both phases (global and regional) of satellite altimetry reductions. Such a procedure is envisioned as a part of the on-going effort, and it will be described in the following chapter and in the next report. In this way, the artificial lowering of the input sigmas can be either completely bypassed, or can be implemented on a much smaller scale than anticipated.

3. TIDAL AND OTHER SEA SURFACE EFFECTS

In following, in principle, the system of classification and description of various sea surface effects by Lisitzin [1974], one can divide these effects into the categories and sub-categories listed below:

1) Astronomical contributions:
   - 1a) Long-period tides,
   - 1b) Diurnal and semidiurnal tides,
   - 1c) Chandler effect,
   - 1d) Variations of the speed of the earth's rotation;

2) Meteorological contributions:
   - 2a) Atmospheric pressure effects,
   - 2b) Wind effects,
   - 2c) Evaporation and precipitation;

3) Oceanographic contributions:
   - 3a) Water density effects,
   - 3b) Currents;

4) Vertical movement of the earth's crust;

5) Melting or forming of continental ice, etc.;

6) Coastal and other local phenomena;

7) Other phenomena.

This study is concerned mostly with the long-period tides and the diurnal and semidiurnal tides of the "astronomical contributions", i.e., with the items 1a) and 1b). However, the Chandler effect, the atmospheric pressure effects, and the water density effects, i.e., the items 1c), 2a), and 3a), could also be considered for a possible inclusion into the SEASAT altimetry adjustment algorithm, most likely in the form of a correction to the altimeter measurements. This approach will be discussed in Section 3.5. On the other hand, the remaining effects, i.e., 1d), 2b), 2c), 3b), 4), 5), 6) and 7), will be deleted from further analysis. They are briefly described in Appendix 4 with an explanation of why an inclusion of these effects in the present altimetry adjustment model would not serve a useful purpose. For the most part, then, the present chapter will be dealing with the long-period, diurnal, and semidiurnal tidal effects; an important part of the analysis will be based on considerations related to the theoretical (equilibrium) tidal model.

3.1 Equilibrium Tide

The first task in this section will be the determination of the tidal effects to be described in the current presentation, and of those which will be described in the next report. The criterion for this organization will be the relative magnitude in the sea level changes due to individual tidal constituents as determined by the equilibrium theory. This theory involves the hypothesis of a completely rigid earth covered over its entire surface (in the absence of continents) by deep water, of no friction in the water envelope as well as of no meteorological or other disturbances.

When determining the height of the theoretical, or equilibrium, tide, its individual component "h_j" is associated with the tidal constituent "j" of amplitude A_j and argument α_j. The total height is then the sum of the individual h_j's. The basic formulas adopted in this development are (129) - (131) of [USCGS,1958] abbreviated here as [US]. Since only the average values of the h_j's with regard to the longitude of the moon's mode are sought at the first stage of the analysis, the "node factor", f, is taken as unity in the pertinent formulas. This implies, for example, that the "permanent tide" symbolized by A_0 is considered through its mean effect over one full revolution (or several full revolutions) of the moon's node; such a revolution is completed in about 18.6 years. The value h_{A_0} in this context will later be related to the development in Section 5.2 of [Blaha, 1980].

In considering the average (in the above sense) combined effect of the moon and the sun, the constituent height can be expressed by

$$h_j = A_j \cos \alpha_j . \tag{1}$$

The amplitude varies with ϕ, the geocentric latitude, as

$$A_j \simeq K_j \, G_j(\phi) \, C_j ; \tag{2}$$

here $f \equiv 1$ is assumed so that the coefficient C_j represents the mean value of a pertinent function with respect to the longitude of the moon's node. In the following, three indices (a, b, c) will be used:

a ... long-period constituents,
b ... diurnal constituents,
c ... semidiurnal constituents.

The meaning of the symbols K_j and $G_j(\phi)$ is thus narrowed down to

$$K_a = \tfrac{1}{2}Ga \simeq 0.13335 \, m , \tag{3a}$$

$$K_b = K_c = Ga \simeq 0.2667 \, m , \tag{3b}$$

where

$$G = (3/4)(M/E)(a/r_M^3) , \tag{3c}$$

with M and E being the moon's and the earth's masses, respectively, a being the mean radius of the earth (6,371 km) and r_M being the mean earth-moon distance; according to the values listed in Table 1 of [US], $G \simeq 0.41865 \times 10^{-7}$. Further,

$$G_a(\phi) = (1 - 3\sin^2\phi) , \tag{4a}$$

$$G_b(\phi) = \sin 2\phi , \tag{4b}$$

$$G_c(\phi) = \cos^2\phi . \tag{4c}$$

In anticipation of the results (i.e., of the constituents to be described in this and the next reports), Table 1 has been constructed featuring the constituent heights expressed according to (1)-(4). Its "extreme magnitude" column features the largest values of h_j which can be reached as follows:

$a \ldots \phi = \pm 90^\circ$,

$b \cdots \phi = \pm 45^\circ$,

$c \cdots \phi = 0$.

In each group (a,b, or c) the constituents are listed in the descending order of magnitude.

In organizing the treatment of the tidal constituents, the decision has been reached to include herein those whose extreme equilibrium magnitude can be as low as 5 cm. If the equilibrium model were realistic, this would indicate the possibility of having a reliable one-decimeter geoidal resolution. In agreement with Table 1, the tidal constituents in this category are: A_0; K_1, O_1; M_2, S_2. The next report will include the constituents whose extreme equilibrium magnitude ranges from 5 cm down to 2.5 cm, which indicates the possibility of a half-decimeter geoidal resolution under the same circumstances. Following the P_1 constituent in the order of importance in the b (diurnal) group would be the Q_1 constituent with C_{Q_1} = .0730 and the extreme magnitude of 0.019m. Similarly, following the K_2 constituent in the c (semidiurnal) group would be the L_2 constituent with C_{L_2} = .0251 and the extreme magnitude of 0.007 m. These and all the other constituents with even smaller magnitudes are left out of consideration. An exception to this statement are the Mm and SSa constituents in the a (long-period) group whose extreme magnitudes in Table 1 are slightly below 2.5 cm. These borderline cases are present because of a very short active life-span of SEASAT (about three months) during which the SSa constituent could result in quasi-systematic influences for most of the altimeter data. The next report will then describe the tidal adjustment enlarged by the constituents: Mf, Mm, SSa; P_1; N_2, K_2. Accordingly, a total of eleven constituents will actively participate in the adjustment of SEASAT altimeter data. They are listed in Table 1 in the order: A_0, Mf, Mm, SSa; K_1, O_1, P_1; M_2, S_2, N_2, K_2.

Even if the water friction as well as meteorological and other disturbances were nonexistent, the equilibrium tidal formulas would be rendered more complex due to the yielding of the earth's crust as a function of its elastic properties. Since the earth's crust is not rigid as originally assumed, it will be deformed during the tidal process. This phenomenon is sometimes called the earth's deformation or the bottom tide, and is given as a multiple of the equilibrium tide. The earth's tidal deformation involves a shifting of mass and changes in the potential, thus causing an additional tidal effect. In agreement with [Bomford, 1975], page 557, these two phenomena will be called here "earth's deformation" and "additional tide". In reality, the situation is complicated by further effects which, however, are relatively small and can be left out of consideration for the present needs.

If the height of the equilibrium tide is denoted AB, in agreement with [Bomford, 1975] one has

$$\text{earth's deformation} = h\,AB\,, \tag{5a}$$

$$\text{additional tide} = k\,AB\,, \tag{5b}$$

where h and k are the Love numbers. As explained in [Vaníček, 1980], geodetic applications have resulted in using the Love numbers h_2, k_2 and in denoting them h, k, respectively. In agreement with this reference the following values are adopted:

$$h \simeq 0.62\,, \tag{6a}$$

$$k \simeq 0.29\,. \tag{6b}$$

In considering satellite altimetry, the tidal effect which can be sensed by the altimeter is a change in the radial distance from the geocenter. If the original equilibrium model is considered in conjunction with the deformable earth (i.e., if the assumption of a completely rigid earth is removed), this effect is given by the equilibrium tide plus the additional tide of (5b), namely

$$\text{"geocentric tide"} = (1+k)\,AB \simeq 1.29\,AB\,. \tag{7}$$

On the other hand, the tide which under these circumstances would be measured by a tide gauge (giving the height of the ocean surface above the deformable ocean bottom) would have the bottom deformation (5a) subtracted from (7), giving

$$\text{"measured tide"} = (1+k-h)AB \simeq 0.67\,AB\,. \tag{8}$$

KIND	SYMBOL	DESCRIPTION	c_j	h_j	EXTREME MAGNITUDE
a	A_0	constant	.7384	$.0985\text{m}\ (1-3\sin^2\phi)$	.197m
	Mf	semimonthly	.1566	$.0209\text{m}\ (1-3\sin^2\phi)\cos\alpha_{Mf}$	.042m
	Mm	monthly	.0827	$.0110\text{m}\ (1-3\sin^2\phi)\cos\alpha_{Mm}$	.022m
	SSa	semiannual	.0728	$.0097\text{m}\ (1-3\sin^2\phi)\cos\alpha_{SSa}$	.019m
b	K_1	declinational luni-solar	.5305	$.1415\text{m}\ \sin 2\phi\ \cos\alpha_{K_1}$	.141m
	O_1	principal lunar	.3771	$.1006\text{m}\ \sin 2\phi\ \cos\alpha_{O_1}$	.101m
	P_1	principal solar	.1755	$.0468\text{m}\ \sin 2\phi\ \cos\alpha_{P_1}$	.047m
c	M_2	principal lunar	.9085	$.2423\text{m}\ \cos^2\phi\ \cos\alpha_{M_2}$	.242m
	S_2	principal solar	.4227	$.1127\text{m}\ \cos^2\phi\ \cos\alpha_{S_2}$	.113m
	N_2	ecliptical lunar	.1759	$.0469\text{m}\ \cos^2\phi\ \cos\alpha_{N_2}$	.047m
	K_2	declinational luni-solar	.1151	$.0307\text{m}\ \cos^2\phi\ \cos\alpha_{K_2}$	.031m

Table 1

Approximate heights of selected tidal constituents, including extreme magnitudes

In combining (7) and (8), one can also write

$$\text{"geocentric tide"} = [(1+k)/(1+k-h)] \times \text{"measured tide"}\ , \qquad (9a)$$

where

$$(1+k)/(1+k-h) \simeq 1.93\ . \qquad (9b)$$

According to [Lisitzin, 1974], page 50, the magnitude of observed long-period tides is, on the average, approximately a 3.7-multiple of the equilibrium value. If h_j^e denotes the long-period "geocentric" tidal elevations with the empirical factor (e) taken into account,

$$e \simeq 3.7 , \tag{10}$$

then a better approximation of the actual situation than that depicted in Table 1 is

$$h_j^e = (1+k)\, e\, h_j \simeq 4.8\, h_j . \tag{11}$$

A somewhat similar outcome -- although more complicated -- could be expected in the case of diurnal and semidiurnal tidal constituents. Instead of merely comparing the amplitudes and phases of the actual and theoretical effects, a solution of dynamic equations would be needed for a realistic representation of tidal phenomena. Be that as it may, the main outcome of this discussion points toward a lower geoidal resolution than was anticipated when only the equilibrium tide (with rigid earth) was considered.

Even if the empirical factor is not as large as indicated in (10), the previously considered resolution capabilities would still be lowered, probably two- or three-fold. Therefore, the practical resolution capabilities are not expected to be at the 5 cm theoretical level as discussed earlier in conjunction with all eleven of the constituents of Table 1 but, rather, at a 10 cm or 20 cm level. However, this still represents a great improvement in geoidal representation when compared with past altimeter data reductions into which the tidal effects were not incorporated at all. For example, the most important of the tidal constituents, M_2, can alone account for over one-meter in elevation effects, when considering its extreme magnitude of Table 1 in conjunction with a factor of the kind (1+k)e adopted in analogy to (11). One can thus conclude that in spite of the actual resolution being coarser than that corresponding to the equilibrium tide (with rigid earth), a meaningful geoidal resolution will nevertheless be improved by an order of magnitude. After all eleven of the constituents of Table 1 have been included in the SEASAT altimetry adjustment model, such a resolution is expected to improve from a 1.5 - 2 m level to about a 20 cm level.

Permanent tide. An approximate equilibrium formula giving the average tide-raising effect, $\overline{N}$, was derived in [Blaha, 1980] and is recapitulated in Appendix 5, equation (A5.1), as

$$\overline{N} = 0.148\,\mathrm{m}\ (\cos 2\phi - 1/3).$$

This formula is refined in (A5.26) to read

$$\overline{N} = 0.147\,\mathrm{m}\,(\cos 2\phi - 1/3). \tag{12a}$$

Because of the identity

$$\cos 2\phi - 1/3 = (2/3)(1-3\sin^2\phi)\ ,$$

the formula can be rewritten as

$$\overline{N} = 0.098\ (1-3\sin^2\phi)\ . \tag{12b}$$

This last result compares well with .0985 m $(1-3\sin^2\phi)$ of Table 1. It also agrees with [Lisitzin, 1974], page 49, where it is implied for the "permanent" tidal potential:

$$W_{A_0} = .96621\ (1\text{-}3\ \sin^2\phi)\ m^2/sec^2\ ;$$

thus

$$W_{A_0}/\overline{g} \simeq .0986\,m\,(1\text{-}3\ \sin^2\phi)\ ,$$

where $\overline{g}$, the average terrestrial gravity, is adopted as 9.80 m/sec².

The basic initial formulas in Appendix 5 will now be related to the development in [Godin, 1972]. The potential induced by the moon is expressed on pages 19, 20 of this reference as

$$V_M \simeq V_2 = (kMa^2/r_M^3)\ P_2(\cos z)\ ,$$

where k is the gravitational constant, $P_2(\cos z)$ represents the Legendre polynomial of order two in cos z, and z is the zenith distance to the moon, the other symbols having been already defined. Since

$$P_2(\cos z) = \frac{1}{2}\ (3\ \cos^2 z - 1) = (3/4)(\cos 2z + 1/3)\ ,$$

one can write

$$V_M \simeq (3/4)(kMa^2/r_M^3)(\cos 2z + 1/3)\ . \qquad (13)$$

Since N_M, the tidal elevation due to the moon, is computed as

$$N_M = V_M/\overline{g}\ ,$$

where $\overline{g}$, the average terrestrial gravity, is

$$\overline{g} \simeq kE/a^2\ ,$$

it follows that

$$N_M \simeq k_1(\cos 2z + 1/3) \ ,$$

$$k_1 = Ga \ ,$$

with G given in (3c). According to the [US] values, $k_1 = 0.2667\,m$ as was already indicated in (3b). The same reference implies that in order to obtain the formula giving N_S for the sun's effect, k_1 should be replaced by $k_2 = 0.4602\ k_1 = 0.1227\,m$. This development for N_M and N_S agrees with (A5.2) together with (A5.3a,b) of Appendix 5.

If a more accurate representation of the permanent tide is sought, the influence of the moon and the sun must be treated separately. For this purpose, the node factor for the moon, denoted in general as f_j, can no longer be assumed to be unity (it is always unity for the sun). When applied separately for the moon and the sun, (2) becomes

$$A_j = K_j\ G_j(\phi)\ C_j\ f_j\ \ldots \text{ moon} \ , \tag{14a}$$

$$A_j' = K_j\ G_j(\phi)\ C_j' \quad \ldots \text{ sun} \ . \tag{14b}$$

The node factor (a function of the longitude of the moon's node with the periodicity of about 18.6 years) changes very slowly from year to year for each constituent. Table 14 of [US] gives the value of the pertinent f_j for the middle of each year between 1850 and 1999. For the permanent tide, the value which corresponds to the active life-span of SEASAT can be associated with mid-year 1978 and is given as

$$f_{A_o} = f_{Mm} = 1.131 \ . \tag{15}$$

If a given constituent represents the moon's action alone, (1) is adopted without change in notations and A_j is computed according to (14a). For a strictly solar constituent, A_j and α_j in (1) are replaced by A_j' and α_j', with A_j' computed as in (14b). If a constituent is composed of both effects, the resulting h_j is obtained as

$$h_j = A_j \cos\alpha_j + A_j' \cos\alpha_j' = K_j G_j(\phi)(C_j f_j \cos\alpha_j + C_j' \cos\alpha_j'). \qquad (16)$$

Since α_j and α_j' are immaterial for the permanent tide, with the aid of (3a) and (4a) equation (16) becomes

$$h_{A_0} = 0.13335\,\mathrm{m}\,(1-3\sin^2\phi)(C_{A_0} f_{A_0} + C_{A_0}') \,. \qquad (17)$$

Table 2 of [US] gives

$$C_{A_0} = 0.5044, \qquad C_{A_0}' = 0.2340 \,,$$

which, when added algebraically, yield the value 0.7384 seen in Table 1 for an average effect. However, in considering the specific case of SEASAT altimetry and the corresponding value f_{A_0} in (15), equation (17) yields the explicit form for the equilibrium height of the constituent A_0:

$$h_{A_0} = 0.1073\,\mathrm{m}\,(1-3\sin^2\phi) \,. \qquad (18)$$

Diurnal constituents K_1 and O_1. The value of h_{K_1} is made up of the moon's and the sun's contribution, hence it is of the form (16). Due to the changing longitude (N) of the moon's node, α_{K_1} differs from α_{K_1}' by a small quantity $-\nu$, namely

$$\alpha_{K_1} = \alpha'_{K_1} - \nu. \tag{19}$$

Similar to the node factor, the periodicity of ν is approximately 18.6 years. Over short periods (e.g. less than a year) it can be considered constant. Table 6 of [US] gives the values of ν according to N. For the beginning of September 1978, the epoch which is quite representative of the SEASAT data series, the value of N found from Table 4 of [US] is

$$N \simeq 178^{0} , \tag{20}$$

implying that

$$\nu \simeq 0.57^{0}. \tag{21}$$

The value in (20) is considered constant for all SEASAT observations.

In order to obtain h_{K_1} from (16) in the form similar to (1) in conjunction with (14a), one has to simplify the following expression:

$$c_{K_1} = C_{K_1} f_{K_1} \cos(\alpha'_{K_1} - \nu) + C'_{K_1} \cos\alpha'_{K_1} , \tag{22a}$$

which corresponds to the quantity inside the parentheses in (16) with K_1 substituted for j and with (19) taken into account. Equation (22a) can be developed into

$$c_{K_1} = (C_{K_1} f_{K_1} \cos\nu + C'_{K_1}) \cos\alpha'_{K_1} + C_{K_1} f_{K_1} \sin\nu \sin\alpha'_{K_1} , \tag{22b}$$

which is of the form

$$C = C_1 \cos\alpha + C_2 \sin\alpha = (C_1^2 + C_2^2)^{\frac{1}{2}} [(C_1 \cos\alpha + C_2 \sin\alpha)/(C_1^2 + C_2^2)^{\frac{1}{2}}] .$$

If ν' is defined as arc tg (C_2/C_1), one has

$$\sin\nu' = C_2/(C_1^2 + C_2^2)^{\frac{1}{2}} , \qquad \cos\nu' = C_1/(C_1^2 + C_2^2)^{\frac{1}{2}} ,$$

and hence

$$C = (C_1^2 + C_2^2)^{\frac{1}{2}} \cos (\alpha - \nu') .$$

When applied to (22b) this yields

$$c_{K_1} = (\tilde{C}_{K_1}\tilde{f}_{K_1}) \cos\tilde{\alpha}_{K_1} , \tag{23a}$$

where

$$\tilde{C}_{K_1}\tilde{f}_{K_1} = (C_1^2 + C_2^2)^{\frac{1}{2}} , \tag{23b}$$

$$C_1 = C_{K_1}f_{K_1} \cos\nu + C'_{K_1} , \qquad C_2 = C_{K_1}f_{K_1} \sin\nu , \tag{23c}$$

$$\tilde{\alpha}_{K_1} = \alpha'_{K_1} - \nu' \qquad , \qquad \nu' = \text{arc tg } (C_2/C_1). \tag{23d}$$

The coefficient $\tilde{C}_{K_1}$ represents a certain mean value defined as

$$\tilde{C}_{K_1} = \text{mean } [(C_1^2 + C_2^2)^{\frac{1}{2}} \cos\nu'] \equiv \text{mean } C_1 , \tag{24a}$$

which in Table 2 of [US] is listed to be

$$\tilde{C}_{K_1} = 0.5305. \tag{24b}$$

Further listed are the values

$$C_{K_1} = 0.3623, \qquad C'_{K_1} = 0.1681.$$

The node factor for K_1 is identical to that for J_1 of equation (76) of [US], and it is listed for mid-1978 as

$$f_{K_1} = f_{J_1} = 0.827.$$

With these values for C_{K_1}, C'_{K_1}, f_{K_1}, and with ν from (21), one calculates (see 23c,d):

$$C_1 \simeq 0.4677, \qquad C_2 \simeq 0.002981, \qquad \nu' \simeq 0.37^\circ;$$

from here it follows (see 23b,d) that

$$\tilde{C}_{K_1}\tilde{f}_{K_1} = 0.4677, \qquad \tilde{\alpha}_{K_1} = \alpha'_{K_1} - 0.37^\circ. \tag{25}$$

The values in (25) could also be obtained more directly from Table 14, and Table 6 or Table 11 of [US], respectively. In particular, 0.4677 could be found upon multiplying 0.5305 in (24b) by the corresponding node factor $\tilde{f}_{K_1}$, listed for mid-1978 in Table 14 (under the heading K_1) as 0.882; and 0.37° could be found, for $N = 178^\circ$, from Table 6 (under the heading ν') or from Table 11 (under the heading K_1). In either case the equilibrium formula (16), applied to K_1 in conjunction with (3b), (4b), (23a) and (25), becomes

$$h_{K_1} = 0.1248\,\mathrm{m}\ \sin 2\phi\ \cos(\alpha'_{K_1} - 0.37^\circ)\ , \tag{26}$$

referring to the epoch of SEASAT altimeter data acquisition. The variable part of the argument, α'_{K_1}, will be described later.

The effect of O_1 is due exclusively to the moon, hence (1) and (14a) apply. In analogy to (19) and the development that followed, α_{O_1} will be written as α'_{O_1} (this, in itself, is immaterial here) plus some small quantity which will again be considered constant due to the short active life-span of SEASAT. In particular,

$$\alpha_{O_1} = \alpha'_{O_1} + (2\xi - \nu) . \tag{27}$$

For N given in (20), Table 4 of [US] yields approximately 0.54° for ξ and 0.57° for ν (see equation 21 above); Table 11 of the same reference yields directly 2ξ-ν under the heading O_1. In either case the result is

$$2\xi - \nu = 0.50^\circ . \tag{28}$$

The coefficient "C" and the node factor for mid-1978 are

$$C_{O_1} = 0.3771, \qquad f_{O_1} = 0.806 . \tag{29}$$

Upon inserting the results (3b), (4b) and (27)-(29) into (1) and (14a), one has the equilibrium formula for O_1 representing the SEASAT observational epoch:

$$h_{O_1} = 0.0811\,\text{m} \ \sin 2\phi \ \cos(\alpha'_{O_1} + 0.50^\circ) . \tag{30}$$

Semidiurnal constituents M_2 and S_2. The most important of all the tidal constituents, M_2, is due exclusively to the moon. It can be developed in a complete analogy to the approach followed for O_1. The argument is written as

$$\alpha_{M_2} = \alpha'_{M_2} + (2\xi - 2\nu) , \tag{31}$$

where ξ and ν were already found; thus

$$2\xi - 2\nu = -0.07^\circ , \tag{32}$$

which is also the value given in Table 11 of [US] under the heading M_2. Further, one has

$$C_{M_2} = 0.9085 \ , \qquad f_{M_2} = 1.038 \ , \tag{33}$$

and

$$h_{M_2} = 0.2515\,\text{m} \ \cos^2\phi \ \cos(\alpha'_{M_2} - 0.07°) \ , \tag{34}$$

which is the equilibrium formula for M_2 corresponding to the SEASAT observational epoch. It has been obtained from (31)-(33) in the same way as (30) was obtained from (27)-(29) except, of course, that (4b) has been replaced by (4c).

The constituent S_2 owes its existence to the sun. The argument is thus written as α'_{S_2} in agreement with the original convention, and the node factor is omitted. In other respects the equilibrium formula for S_2 is derived similar to (34) above, namely

$$h_{S_2} = 0.1127\,\text{m} \ \cos^2\phi \ \cos\alpha'_{S_2} \ . \tag{35}$$

In this case, no special considerations related to the SEASAT observational epoch are necessary.

3.2 Tidal Arguments

The explicit expressions for the equilibrium tidal arguments will be developed in a way similar to Table 2 of [US] with a few minor changes. One change pertains to the constant part "u" which is presently expressed numerically (in °) and represents the SEASAT observational epoch. With regard to the computation of α_j', the variable part of the argument (in Table 2 of [US] denoted as V), UT ±12 hours is used instead of T, the hour angle of the mean sun at Greenwich at the time of the tidal evaluation. Since all the quantities will be considered as given in degrees instead of hours, the following applies:

$$T = UT \pm 180^\circ , \tag{36}$$

where UT (in °) is obtained by multiplying UT (in hours) by 15 (°/hour), etc. The other two variables needed for the evaluation of α_j' at Greenwich for the presently discussed constituents are h, the mean longitude of the sun, and s, the mean longitude of the moon. In terms of local -- rather than Greenwich -- arguments, UT is replaced by UT + λ, where λ (in °) is the customary east longitude of the point where the tidal evaluation is sought, symbolized by

$$\text{local argument} \ldots UT \rightarrow UT + \lambda . \tag{37}$$

In order to indicate the computation of the equilibrium tidal arguments at Greenwich, Table 2 has been constructed listing these arguments in two parts (see its second column), α_j' and u_j; the final argument is

$$\alpha_j = \alpha_j' + u_j . \tag{38}$$

CONSTI-TUENT	GREENWICH ARGUMENT α'_j	$+u_j$	SPEED ^{0}/day	^{0}/hour	α'_j Jan 0.5, 1900	PERIOD
K_1	UT+h+90^0	-0.37^0	360.985647335	15.04106864	189.696678^0	23.9345 hr
O_1	UT-2s+h-90^0	+0.50^0	334.632853797	13.94303557	188.821833^0	25.8193 hr
M_2	2UT-2s+2h	-0.07^0	695.618501132	28.98410421	18.518511^0	12.4206 hr
S_2	2UT	+0.00	720.	30.	0.	12 hr

Table 2

Greenwich arguments and related quantities
for selected equilibrium tidal constituents

The α'_j part agrees with [Schwiderski, 1980], page 172, and with [Lisitzin, 1974], page 12.

For the explicit computation of α'_j, the expressions for h and s are adapted from [US], page 162, as

$$h = 279.696678^0 + 36{,}000.768925^0 T + 0.000303^0 T^2 , \qquad (39a)$$

$$s = 270.437422^0 + 481{,}267.892000^0 T + 0.002525^0 T^2 + 0.000002^0 T^3 , \qquad (39b)$$

where T is the number of Julian centuries (of 36,525 days) reckoned from January 0.5, 1900 at Greenwich, i.e., from December 31, 1899, 12h UT. For January 0.0, 1978 at Greenwich, the value of T is 28,488.5/36,525; upon considering (39 a,b) one has

$$[h] = 279.310976^0 , \qquad [s] = 166.218322^0 , \qquad (40)$$

where the brackets have been used to indicate this specific time epoch.

Near a point of expansion, i.e., certainly within a year, h and s can be considered as linear functions of time and their speeds in ^{0}/day, etc., can be evaluated using the terms linear in T in (39 a,b). When considered together with (40), these speeds make it possible to compute h and s for any instant in 1978 accurately as

$$h = 279.310976^0 + 0.985647335^0 \cdot D + 0.04106864^0 \cdot hr + 0.00068448^0 \cdot min + 0.00001141^0 \cdot sec\ , \quad (41a)$$

$$s = 166.218322^0 + 13.176396769^0 \cdot D + 0.54901653^0 \cdot hr + 0.00915028^0 \cdot min + 0.00015250^0 \cdot sec\ , \quad (41b)$$

where D = day number in 1978, and hr, min, sec represent hours, minutes, seconds in UT for that day. From the formulas (41 a,b) the various rates of change in h and s are apparent. They also confirm the periodicity of h (365.2421988 days = tropical year) and of s (27.32158164 days = tropical month).

The numerical values of α'_j at Greenwich for any instant in 1978 can be found from the general expression appearing in the second column of Table 2. The rate of change in UT, taken in the interval 0-24 hours, is 15^0/hr, 0.25^0/min and 0.00416667^0/sec, while the initial values and the rates of change in the other two variables, h and s, have been given in (41 a,b). The required combinations of UT, h and s thus yield

$$\alpha'_{K_1} = 9.310976^0 + 0.985647335^0 \cdot D + 15.04106864^0 \cdot hr + 0.25068448^0 \cdot min + 0.00417807^0 \cdot sec\ , \quad (42a)$$

$$\alpha'_{O_1} = 216.874331^0 - 25.367146203^0 \cdot D + 13.94303557^0 \cdot hr + 0.23238393^0 \cdot min + 0.00387307^0 \cdot sec\ ; \quad (42b)$$

$$\alpha'_{M_2} = 226.185307^0 - 24.381498868^0 \cdot D + 28.98410421^0 \cdot hr + 0.48306840^0 \cdot min + 0.00805114^0 \cdot sec \ , \qquad (43a)$$

$$\alpha'_{S_2} = 30.^0 \cdot hr + 0.5^0 \cdot min + 0.00833333^0 \cdot sec \ , \qquad (43b)$$

where D, hr, min, sec were defined following (41 a,b). From these formulas the rates of change in the arguments α'_j and thus also α_j are apparent and agree with Table 2 of [US] wherever they are comparable (i.e., they agree with the values printed in [US] as "speed per solar hour" which, however, exhibit fewer significant digits than the speeds derived above). Furthermore, these rates also agree with [Estes, 1980], page 118, and with [Godin, 1972], page 232; they agree approximately with [Schwiderski, 1980], page 172, [Estes, 1980], page 101, and [Lisitzin, 1974], page 12. The rates associated with "D" and "hr" are further presented in Table 2, columns 3 and 4, respectively, under the headings ^{0}/day and ^{0}/hour.

The fifth column of Table 2 lists α'_j at Greenwich for January 0.5, 1900 obtained, with the aid of the second column, from (39 a,b) for T=0. One could evaluate α'_j at any instant also from these values upon applying the rates listed in the columns 3 and 4. However, this would lead to a slight loss of accuracy even if sufficient digits are used in the arithmetic, due to neglecting the terms in T^2 (and T^3) inherent in the formulas for h and s in (39 a,b). By comparison, the terms in T^2 and T^3 did enter (41 a,b) and thus also (42 a,b) and (43 a,b) developed herein in view of SEASAT altimetry. The latter formulas are advantageous to use not only for their accuracy, but also because they are very simple and do not necessitate a large number of significant digits for their evaluation.

3.3 Adjustment Using Equilibrium Formulas

The basic model equation of satellite altimetry was written in equation (3.1) of [Blaha, 1979] as

$$H = R - r + d \; , \tag{44}$$

where H represents the altimetry, R is the distance from the geocenter to the satellite at the time of observation, d is a correction, always smaller than 5 m for the satellite altitude under 1,000 km as described e.g. on page 28 of [Blaha, 1977] or in [Blaha, 1977'], and r is the distance from the geocenter to a sub-satellite point on the sea surface; it is given on page 15 of [Blaha, 1979] as

$$r = r' + N \; , \tag{45}$$

where r' is the corresponding distance to the (geocentric) reference ellipsoid and N represents the geoid undulation. The main feature of an earlier approach consisted in expressing N (and thus r) in terms of the geoidal parameters only, as if the measured sea surface coincided with the geoid. Although this model deficiency was of little consequence in past adjustments of GEOS-3 altimetry, it will be removed from the SEASAT altimetry model by separating N into two parts:

$$N = N' + N'' \; , \tag{46}$$

where N' is expressed in terms of the geoidal parameters as before, but where N'' now represents the separation between the geoid and the measured sea surface.

If the separation were to be expressed with the aid of the equilibrium formulas considered presently, one would write

$$N'' = h_{A_0} + h_{K_1} + h_{O_1} + h_{M_2} + h_{S_2} \; , \tag{47}$$

with the following notations (corresponding so far to $\lambda = 0$):

$$h_j = A_j \cos \alpha_j \; , \tag{48}$$

$$\alpha_j = \alpha_j' + u_j \quad ,$$

where α_j' and thus α_j are Greenwich arguments. For the permanent tide, one can adopt both the local and the Greenwich arguments as

$$\alpha_{A_0} \equiv 0 \; . \tag{49}$$

According to (37) and to the second column of Table 2, the other pertinent local arguments (allowing for any λ) are

$$\alpha_{K_1} = \alpha_{K_1}' + \lambda - 0.37^\circ \; , \tag{50a}$$

$$\alpha_{O_1} = \alpha_{O_1}' + \lambda + 0.50^\circ \; ; \tag{50b}$$

$$\alpha_{M_2} = \alpha_{M_2}' + 2\lambda - 0.07^\circ, \tag{51a}$$

$$\alpha_{S_2} = \alpha_{S_2}' + 2\lambda \; . \tag{51b}$$

In their general form, the quantities α_j' appear in the second column of Table 2; for their numerical evaluation one can take advantage of equations (42 a,b) and (43 a,b) which have been tailored for the use with SEASAT altimetry.

The amplitudes A_j for the equilibrium constituents A_o, K_1, O_1, M_2 and S_2 can be adopted, with a modification to be explained, from equations (18), (26), (30), (34) and (35), respectively:

$$A_{A_o} = c \cdot 0.1073\,\mathrm{m}\ (1-3\sin^2\phi)\ ; \tag{52}$$

$$A_{K_1} = c \cdot 0.1248\,\mathrm{m}\ \sin 2\phi\ , \tag{53a}$$

$$A_{O_1} = c \cdot 0.0811\,\mathrm{m}\ \sin 2\phi\ ; \tag{53b}$$

$$A_{M_2} = c \cdot 0.2515\,\mathrm{m}\ \cos^2\phi\ , \tag{54a}$$

$$A_{S_2} = c \cdot 0.1127\,\mathrm{m}\ \cos^2\phi\ . \tag{54b}$$

If strictly the equilibrium tide in conjunction with a rigid earth were considered, the above constant c would be

$$c \equiv 1\ . \tag{55a}$$

If the equilibrium formulas should allow for the earth's deformation, according to (7) one could adopt

$$c = 1 + k \simeq 1.29\ . \tag{55b}$$

If, in addition, discrepancies between the actual and theoretical tidal magnitudes should be taken into account, equation (11) suggests the adoption of

$$c = (1+k)e \simeq 4.8\ . \tag{55c}$$

It should be noted that in practical adjustments this (or a similar) factor will eventually be used only in conjunction with the long-period constituents for which it was originally deduced (see equation 10 and the text preceding it). However, at this point one can adopt (55c) as a practical value for evaluating all of (52), (53 a,b) and (54 a,b).

It follows from the foregoing that if the altimeter adjustment should take into account the tidal elevation N" from (47), and if the equilibrium model in some form should serve for this purpose, the individual values of h_j could be computed from (48) with A_j given in (52), (53 a,b), (54 a,b), and with α_j computed as specified in (49), (50 a,b), (51 a,b) and in the text that followed. A practical value for c could be adopted from (55c). Such N" could then serve as a simple correction to be added to the value of N' computed customarily through geoidal parameters. However, as will be described in the next paragraph, one can go one step further and consider the tidal amplitudes as adjustable quantities. For example, a set of parameters (point-mass magnitudes) present in a regional adjustment of satellite altimetry could be augmented by five parameters representing the A_0, K_1, O_1, M_2 and S_2 constituents. It would not then be crucial whether or not c from (55c) is accurate; the adjustment itself would provide the corrections to tidal amplitudes which, in effect, would produce separate c's for the individual tidal constituents in the region of interest.

The motivation for an adjustment of tidal amplitudes, A_j, is offered on page 25 of [Lisitzin, 1974]:

Charts representing such surveys for a given tidal constituent, for example M_2 or K_1, contain the corresponding co-tidal lines, which join all the points for which high water of the constituent concerned is obtained at the same time. The distribution of the amplitudes in the oceans, represented by the tidal co-range lines characteristic of a given amplitude, is a task which is still more difficult.

The above statement concerns especially the diurnal and semidiurnal constituents. However, the motivation for adjusting also the amplitudes of the long-period constituents is apparent from the presence of an approximate average factor e in (10), indicating that the actual amplitudes could be much larger than those found from the theory. In adopting the model equation (48), namely

$$h_j = A_j \cos \alpha_j , \tag{56}$$

with the appropriate A_j given in (52)-(54b) together with c in (55c), and the α_j given in (49)-(51b), etc., the adjusted h_j is

$$h_j^a = h_j + \Delta A_j \cos \alpha_j ,$$

or

$$h_j^a = h_j P_j + h_j , \tag{57a}$$

$$P_j = \Delta A_j / A_j , \tag{57b}$$

where P_j is an (adjusted) parameter; the adjusted amplitude is

$$A_j^a = A_j + \Delta A_j \equiv A_j (1 + P_j) . \tag{57c}$$

From (57a) it is clear that this type of extension to the altimetry model is very simple to perform. According to (47), the adjusted N" is

$$N''^{a} = h_{A_o}P_{A_o} + h_{K_1}P_{K_1} + h_{O_1}P_{O_1} + h_{M_2}P_{M_2} + h_{S_2}P_{S_2}$$
$$+ (h_{A_o} + h_{K_1} + h_{O_1} + h_{M_2} + h_{S_2}) \ , \qquad (58)$$

where the quantity inside the parentheses could be called a "correction"; if there is no adjustment contemplated, this correction would indicate, very approximately, the separation between the geoid and the sea surface at the place of observation. The formula (58) represents a part of an observation equation to be joined to the corresponding expression for N' and, eventually, to the altimeter observation equation along the lines of (44), (45) and (46). In particular, $-(h_{A_o} + h_{K_1} + h_{O_1} + h_{M_2} + h_{S_2})$ should be added to the previously computed constant term and $[-h_{A_o}, -h_{K_1}, -h_{O_1}, -h_{M_2}, -h_{S_2}]$ should augment the row of the pertinent observation equation, while the column-vector $[P_{A_o}, P_{K_1}, P_{O_1}, P_{M_2}, P_{S_2}]^T$ should augment the column of the parameters entering the adjustment. Each of the new parameters (P_{A_o}, etc.) can be weighted at its initial value, in this case the zero value. A "loosely" weighted parameter corresponds, for example, to

$$\sigma_{P_j} = 1, \qquad (59)$$

which indicates, according to (57b), that the sigma attached to ΔA_j could be quite large, in particular, that it could have the magnitude of the amplitude itself. In some applications one could require that the amplitude factor $(1 + P_j)$, and thus P_j, should be the same for certain constituents. This will be explained in connection with a more practical adjustment model to be considered next.

3.4 Practical Tidal Adjustment

The tidal adjustment described in the last section could be adopted for SEASAT data reductions if the equilibrium model well approximated the actual situation, except perhaps for the amplitude factor $(1+P_j)$. Unfortunately, this is not the case in general. The actual diurnal and semidiurnal constituents show large deviations from such a model in more than one respect. Nevertheless, the past development is useful for the long-period tidal adjustment. Presently, this statement applies to A_0 so that (57a) reads

$$h^a_{A_0} = h_{A_0} P_{A_0} + h_{A_0} , \tag{60a}$$

where, in agreement with (49), (52) and (55c) ,

$$h_{A_0} = 0.515\,\mathrm{m}\,(1-3\sin^2\phi) . \tag{60b}$$

One notes that the incorporation of A_0 in some form is important because the permanent tidal deformation is not included in the J_2 coefficient of the earth's gravity field (the dynamic form factor of the earth).

With regard to the diurnal and semidiurnal constituents, a solution of the dynamic equations leads to a more realistic representation of the tidal phenomenon than in the case of the equilibrium model. As outlined in [Lisitzin, 1974], pages 24 and 25, these equations (5, 6 and 7 in this reference) take into account, in addition to the astronomical tide-generating forces, also the Coriolis force, the existence of continents, the effects of bottom friction, viscosity, etc. Eventually, the solution of these equations (they correspond essentially to equations A1 in [Estes, 1980]) results in the determination of spherical-harmonic tidal coefficients. For a given tidal constituent, these coefficients lead to an expansion similar to equation (3) of [Estes, 1980]. This formulation is described with the aid of the following model:

$$\xi_j = A_j \cos(\alpha_j - \psi_j) \equiv A_j \cos\psi_j \cos\alpha_j + A_j \sin\psi_j \sin\alpha_j, \tag{61}$$

where

ξ_j = constituent height observable by tidal gauges,

α_j = Greenwich argument of the constituent,

$A_j \equiv A_j(\phi,\lambda)$ = amplitude of the constituent,

$\psi_j \equiv \psi_j(\phi,\lambda)$ = phase angle of the constituent.

The longitude (λ) of the place of observation is not explicitly needed in α_j since it is included in ψ_j. The angles α_j are thus computed as in (50a)-(51b), except that λ in these equations is suppressed.

Equation (61) is reformulated to read

$$\xi_j = a_j \cos\alpha_j + b_j \sin\alpha_j , \tag{62a}$$

where

$$a_j \equiv a_j(\phi,\lambda) = A_j \cos\psi_j = \sum_n \sum_m (a_{j_{nm}} \cos m\lambda + b_{j_{nm}} \sin m\lambda) P_{nm}(\sin\phi), \tag{62b}$$

$$b_j \equiv b_j(\phi,\lambda) = A_j \sin\psi_j = \sum_n \sum_m (c_{j_{nm}} \cos m\lambda + d_{j_{nm}} \sin m\lambda) P_{nm}(\sin\phi), \tag{62c}$$

from which it follows that

$$A_j = (a_j^2 + b_j^2)^{\frac{1}{2}} , \tag{62d}$$

$$\cos\psi_j = a_j/A_j , \qquad \sin\psi_j = b_j/A_j . \tag{62e}$$

In these formulas $a_{j_{nm}}$, etc., are the spherical-harmonic tidal coefficients of degree and order (n,m) associated with the constituent j, and $P_{nm}(\sin\phi)$ are the (conventional) Legendre functions in the argument $\sin\phi$, ϕ being the geocentric latitude.

The above model will now be related to the one featuring h_j in the role of the "geocentric tide". This h_j behaves approximately as indicated in (9a). Further analysis of this relationship will be presented in the next report. When applying (9a) to the individual constituents, one obtains

$$h_j = c' \xi_j , \tag{63a}$$

where

$$c' = (1+k)/(1+k-h) \simeq 1.93 . \tag{63b}$$

The adjustment model thus becomes

$$\begin{aligned} h_j = c' \xi_j &= c'(a_j \cos\alpha_j + b_j \sin\alpha_j) \\ &= c'(A_j \cos\psi_j \cos\alpha_j + A_j \sin\psi_j \sin\alpha_j) . \end{aligned}$$

If dA_j is the correction to A_j, one has

$$h_j^a = h_j + c' \, dA_j(\cos\psi_j \cos\alpha_j + \sin\psi_j \sin\alpha_j) ,$$

or, in analogy to the development in the last section,

$$h_j^a = h_j P_j + h_j , \tag{64a}$$

$$P_j = dA_j / A_j , \tag{64b}$$

$$A_j^a = A_j + dA_j \equiv A_j(1+P_j) , \tag{64c}$$

where h_j is computed as

$$h_j = c'(a_j \cos\alpha_j + b_j \sin\alpha_j) ; \tag{64d}$$

a_j, b_j were defined in (62 b,c), c' was given in (63b) and α_j was described following (61).

Upon collecting the results (including the A_0 constituent in 60 a,b), one notices that the adjustment could proceed exactly as outlined in (58) and the text that followed. However, the diurnal and semidiurnal constituents could now be associated with a smaller sigma than that of (59) because the present model is likely to describe the actual situation much closer than the previous one. Although (59) could be still used in conjunction with A_0, the sigma associated with K_1, O_1, M_2, S_2 could be adopted, for example, as

$$\sigma_{P_j} = 0.5 . \tag{65}$$

This quantity is again associated with the zero initial value of the parameter P_j. A statement made previously with regard to c in (55) can now be repeated for c' in (63b). In particular, it is not important that this quantity be exceedingly accurate because the adjustment will provide a correction to the tidal amplitude for each individual constituent, resulting in the amplitude factor $(1+P_j)$ which can directly compensate for a possible deficiency in c'.

In the above adjustment, one might wish to stipulate that the amplitude factor for certain constituents should be the same. Such a constraint would necessarily lower the flexibility of the tidal adjustment and thus may not be exercised in practice. Be that as it may, the constraint would stipulate that the corresponding P_j's are equal, which can be achieved through an observation equation attributed a large weight. For example, if this should be done for the semidiurnal constituents M_2 and S_2, one would generate the following observation equation where v_P is the residual:

$$v_P = P_{M_2} - P_{S_2} + 0 , \tag{66a}$$

$$\sigma_P = \text{small (ex.: 0.01)} . \tag{66b}$$

If the equality between P_{M_2} and P_{S_2} above were not a stringent requirement but a property to be satisfied only approximately, the sigma in (66b) would be made larger.

The next report will describe a useful extension to the practical tidal adjustment in that also the phase angle, in addition to the amplitude, will be considered adjustable. Each constituent will then have associated with it, independent of the geographic location, one amplitude factor and one phase correction. Since the model is nonlinear insofar as the phase angle is concerned, the smallness of the corrections to both parameters (P_j, ψ_j) will become important. This means that the adjustment model will have to be reasonably accurate, which will warrant a discussion with regard to the coefficient c', as well as to ocean loading effects and other phenomena.

3.5 Possible Inclusion of Other Sea Surface Effects in the Satellite Altimetry Model

Some sea surface effects which are not due to the tide-generating forces of the moon and the sun, but which could be included in the presently discussed satellite altimetry model will be now briefly described.

Chandler effect. This category encompasses the variations in sea level due to the motion of the true celestial pole (the instantaneous axis of the earth's rotation) with respect to the earth's crust. The character of the sea level changes is similar to that of long-period tidal constituents, the period being now about 14 months. According to [Lisitzin, 1974], page 52, the value of ΔV, the potential of the deforming force, is

$$\Delta V = -\tfrac{1}{2}\omega^2 a^2 (x \cos\lambda + y \sin\lambda) \sin 2\theta ,$$

where ω is the angular speed of the earth's rotation, a is the average radius of the earth, θ and λ are the co-latitude and longitude of the point of interest, respectively, and x, y are the customary rectangular coordinates of the instantaneous pole with respect to an average position. In order to obtain an equilibrium formula under the hypothesis of rigid earth, the above value would be divided by $\bar{g}$, the average gravity. As in equation (7), a "geocentric tide" formula corresponding to a deformable earth would follow upon multiplying this result by $(1+k)$. Similar to (11), an "empirical" formula can be obtained by multiplying the latter by a factor "e", suggested in [Lisitzin, 1974] to be, in this case, about 5 on the average. It appears that the maximum amplitude computed with the empirical formula may reach a few centimeters. Because of this small magnitude and because of a relatively short life-span of SEASAT the Chandler effect has not been considered as subject to adjustment. It could be, however, translated into a correction to altimeter measurements if one chooses to take this effect into account.

Atmospheric pressure effects. The sea level reacts, in principle, to changes in atmospheric pressure like a reverse barometer. When the pressure rises the sea level decreases and vice-versa. In addition to local and short-term variations which are not considered here, there are seasonal variations of this kind that can be significant. In fact, due to the short life-span of SEASAT which thus functioned essentially only during "one season", neglecting these variations could result in systematic errors in global as well as local geoid determinations. Fortunately, seasonal or even monthly maps of sea level changes due to this effect exist. One or more of these maps can be digitized and a correction, to be applied to an altimeter measurement, can be estimated according to the location of the observation point (and the date, in case an appropriate monthly map should be selected by the computer).

The needed positional information can be computed as a matter of course either before or during the altimetry adjustment process. The data stored on magnetic tapes include the arc's ID from which the date and time for the epoch are obtained, as well as the arc's state vector (s.v.) parameters, the time of each measurement with respect to the epoch, and the measurement itself. From the s.v. parameters and the time of each event the foot-point latitude and longitude are computed in the orbital integrator. Since the geographic coordinates are needed only very approximately for this purpose, a reduced set of (2,2) spherical-harmonic coefficients could be employed if one decided to apply the correction beforehand, at some preprocessing stage.

Water density effects. The most important sea level effects considered in this category are due to water temperature and salinity, which may vary from ocean to ocean. Even if one could eliminate or determine mathematically all the significant variations in the sea surface, such as the astronomical effects, various seasonal and local effects, etc., the resulting "static" sea surface would not coincide with the static geoid as the datum of height. In particular, this surface would not be equipotential. Since the spherical-harmonic (S.H.) model of satellite altimetry implicitly assumes such an equipotential surface, a least-squares adjustment would result in certain deformations to both the S.H. potential coefficients and the altimeter observations, even if they were errorless a-priori. In other words, one would be in the presence of a modeling error which the adjustment would accommodate in the least-squares sense.

The departure of such a "static" sea surface from an equipotential surface comes to light when the mean sea level (MSL) is scrutinized in terms of geopotential numbers. The geopotential numbers serving to express potential differences are usually measured in geopotential units, g.p.u. (1 g.p.u. = 1 kgal meter). The g.p.u. correspond to within 2% to the height above the geoid in meters; in particular, 1 g.p.u. corresponds to 1.02 m. Along these lines, the MSL is 0.25 g.p.u. "higher" in the northern hemisphere than in the southern hemisphere. With the above definition in mind, one can refer to one ocean as being higher than another in linear units. This difference of 0.25 m could be due to the fact that in the South Pacific Ocean water contains more salt than in the North Pacific Ocean and, further, that the North Atlantic Ocean is warmer than the South Atlantic Ocean. Additional differences in height exist. Because of the water density differences, the Pacific Ocean is on the average 0.7 m higher than the Atlantic Ocean. But since the highest MSL occurs in the western parts of all oceans, the difference across the Panama canal is only about 0.2 m. The Indian Ocean lies approximately between these two oceans insofar as the heights are concerned. Toward the poles there is a decrease in height. The MSL in adjacent and Mediterranian-type seas is generally lower than in the oceans; for example, the difference in the MSL between the Atlantic Ocean and the Mediterranian Sea is about 0.3 m, concentrated around the Straights of Gibraltar.

It appears that the geoidal results can improve if this knowledge is digitized and, subsequently, applied in the form of a correction to altimeter measurements. The positional information needed for the evaluation of this correction is readily available, as already explained in connection with the atmospheric pressure effects. The size of this correction would be chosen in such a way as to make the geoid correspond to an average geopotential number associated with the MSL. In some oceans the departure of the "static" sea surface from an equipotential surface may not be sufficiently well known. It may then be desirable to subject it to a sea surface slope adjustment and, eventually, a vertical shift adjustment, following the approach outlined in Section 5.3 of [Blaha, 1980].

In addition to an average effect of water temperature and salinity discussed above, seasonal effects are also of interest. The cause of the most important sea level changes in this category are the temperature changes. According to recent results, the average ranges of sea level variations in this group are on the order of 11 cm, and maximum ranges may reach about 25 cm in the regions north of Bermuda. The changes due to the salinity effect are usually small, ranging under 5 cm. In order to compute appropriate corrections, the method of digitized maps (monthly, etc.) could be employed either for each effect separately or for both effects together.

4. CONCLUSIONS

Several improvements recently designed for the satellite altimetry adjustment algorithm were described in Chapter 2, dealing specifically with a global adjustment of SEASAT observations. A criterion was presented stipulating that the length of individual satellite arcs should not exceed 7 minutes in duration. Its aim is to counter the inherent modeling error of the short-arc mode felt mainly at satellite points far from the arc's epoch, due to the presence of errors (including the truncation) in the spherical-harmonic (S.H.) potential coefficients entering the orbital integrator as constants. Another criterion was designed with regard to a minimum length of a satellite arc. It was suggested that only the arcs of at least 50.4 seconds in duration (or 3^0 in angular measure) should enter the adjustment since shorter arcs could absorb some of the geoidal detail through the corrections to the state vector (s.v.) parameters. The ground tracks of arcs exceeding 50.4 seconds in duration will intersect, in general, with ground tracks of three or more crossing arcs and will assure a cantelever effect in the geoidal adjustment thereby preventing undue deformation.

A practical feature providing for a reasonable selection of observational density along a pass was developed, indicating that every 8th observational point entering the adjustment is sufficient to represent adequately SEASAT capabilities. In this way, the separation between measurements along the tracks is $\frac{1}{4}^0$ while the (fixed) separation across the tracks is 1^0. This allows for both sufficient filtering (adjustment) in the observations and sufficient detail in case geoidal profiles are to be plotted along the ground tracks. Another feature, giving rise to the so-called "observed" geoid, was designed specifically for such plotting. It uses the original altimeter measurements in combination with the adjusted s.v. parameters. The "observed" geoid thus contains the high-frequency information present in the measurements, but is improved overall through the reduction of orbital errors achieved in the global adjustment.

Significant savings in terms of computer run-time were achieved through a reduction in the number of constants entering the orbital integrator. Originally, these constants corresponded to a given (truncated) set of S.H. coefficients used in the geoidal adjustment. However, for the needs of the orbital integrator (only), this set can be further truncated, for example, from the original (14,14) set to a new (10,10) set. This means that instead of 225 constants merely 121 constants are used to compute the satellite positions. The criterion guiding this development was the need to preserve the high quality of SEASAT altimetry. It was concluded that additional truncations to an (8,8) set (i.e., down to 81 constants) would also be possible in some cases.

A consideration was given to the possibility of reducing the number of s.v. parametersfrom six to four per orbital arc. However, it was concluded that only insignificant computer savings would be realized while, on the other hand, the rigor of the short-arc altimetry model would be compromised, especially for eccentric satellite orbits. The original short-arc algorithm was therefore retained without modifications.

During past GEOS-3 data reductions the tidal effects were neglected due to the one-meter level of observational noise. In conjunction with SEASAT whose noise, ephemeris, etc., are greatly improved with respect to the GEOS-3 system, these effects can no longer be disregarded in general. The tidal effects can then be considered in the first (global) adjustment, in the second (regional) adjustment designed to model the residuals from the first adjustment via point-mass parameters, or in both adjustments. If they were to be considered only in the regional adjustment of SEASAT altimetry, they should be fully reflected in the residuals from the global adjustment. However, this might not be possible in many cases because of the relatively weakly constrained s.v. parameters which could easily absorb some of the tidal effects into their vertical component (one-sigma in the vertical direction of the satellite ephemeris used is about 1.6 m). One could circumvent at least a part of this problem by artificially decreasing the positional sigmas of the s.v. parameters at a latter stage of the global adjustment in which essentially the residuals alone would be affected. Such a process would be contingent, among other things, upon an excellent quality of the SEASAT observational system (this was indeed confirmed in recent real data reductions). This option is available in the adjustment algorithm, but a need for it will be less extensive than what might have been anticipated. The reason for this stems from the methodology adopted for the development of tidal adjustment, which is to become a part of both the global and the regional SEASAT altimetry adjustment.

The tidal and other sea surface effects are treated in Chapter 3. They are first divided into the following categories:

Astronomical contributions: long-period tides, diurnal and semidiurnal tides, Chandler effect, variations of the speed of the earth's rotation;

Meteorological contributions: atmospheric pressure effects, wind effects, evaporation and precipitation;

Oceanographic contributions: water density effects, currents;

Vertical movement of the earth's crust;

Melting and forming of continental ice, etc.;

Coastal and other local phenomena;

Other phenomena.

The most important sea surface effects for the present analysis are the long-period tides and the diurnal and semidiurnal tides of the above astronomical contributions; these will eventually comprise the tidal adjustment within the SEASAT altimetry adjustment. Of the other effects, only the Chandler effect, atmospheric pressure effects, and water density effects were considered in Chapter 3, mostly in the form of a correction which could be applied to SEASAT altimeter observations. The remaining effects, briefly described in Appendix 4, were eliminated altogether from consideration.

The basis for treatment of tidal effects in Chapter 3 was provided by the theory of equilibrium tides. According to their relative importance, the tidal effects were divided into two groups, the first described herein and the second to be described in the next report. The first group includes the tidal constituents denoted as A_0, K_1, O_1, M_2, S_2, while the second group includes the constituents symbolized by Mf, Mm, SSa, P_1, N_2, K_2. Eventually, 11 tidal constituents will be incorporated into the SEASAT altimetry adjustment, recapitulated as follows:

Long-period: A_0(constant), Mf(semimonthly), Mm(monthly), SSa(semiannual);

Diurnal: K_1(declinational luni-solar), O_1(principal lunar), P_1(principal solar);

Semidiurnal: M_2(principal lunar), S_2(principal solar), N_2(ecliptical lunar),K_2(declinational luni-solar).

The tidal amplitude was considered to be the most important element in a tidal adjustment. An adjustment algorithm for this quantity was described, in Chapter 3, both with regard to the equilibrium tidal model and to a "practical" tidal model; the latter is given in terms of spherical-harmonic tidal coefficients obtained from a solution of the "dynamic equations". The tidal arguments are the same in both these models. An efficient algorithm for their computation was given in terms of the Universal Time (U.T.), in addition to the date and the geographic coordinates of the point under consideration. The tidal model where the phase angle is also subject to adjustment will be described in the next report. In it, the ocean loading effects and the relation between the "geocentric tide" sensed by the altimeter and the "measured tide" sensed by a tidal gauge will also be discussed, as well as an efficient way to incorporate the "practical" tidal model into the present satellite altimetry adjustment algorithm.

APPENDIX 1

NOTE ON A GLOBAL R.M.S. MISCLOSURE OF SEASAT ALTIMETRY

The parameters in a global adjustment of SEASAT altimetry are the spherical-harmonic (S.H.) potential coefficients and the state vector (s.v.) parameters. In the real data reductions described in [Hadgigeorge et al., 1981], the former consisted of a (14,14) subset from the Goddard Earth Model GEM 10 and the latter were supplied by the NSWC precise ephemeris. The reference (normal) gravity field parameters were those recommended by IUGG/IAG [1975]. Both adjustable groups of parameters (S.H., s.v.) in the global phase were weighted according to their reliability, while the reference field parameters were taken essentially as fixed (either completely fixed or heavily weighted).

However, in order to ascertain the influence of various factors in the SEASAT altimetry on the total model variance defined below as σ^2 total, only the misclosures (constant terms) in the observation equations are used; the total model variance is represented by the r.m.s. misclosure as gathered from all the SEASAT passes whose length was restricted to 7 minutes in duration. The basic relationship in this analysis with self-explanatory notations is presented as follows:

$$\sigma^2_{\text{total}} = \sigma^2_{\text{truncation}} + \sigma^2_{\substack{\text{ephemeris}\\ \text{(radial dir.)}}} + \sigma^2_{\substack{\text{altimetry}\\ \text{(noise)}}} + \sigma^2_{\substack{\text{terr. param.}\\ \text{(S.H., ref.)}}} + \sigma^2_{\substack{\text{algorithm}\\ \text{(short-arc)}}}$$

The theoretical error caused by the truncation beyond the (14,14) set of S.H. coefficients is obtained from the covariance function (this function serves to compute degree variances for geophysical quantities, here geoid undulations). The sigma, computed with the aid of equations (10) and (25a) and Table 7 of [Tscherning and Rapp, 1974], is

$$\sigma_{truncation} = 4.887\,m .$$

The sigma for the ephemeris (in the radial direction) has been given as

$$\sigma_{ephemeris} = 1.6\,m .$$

The sigma associated with the altimeter noise for SEASAT is usually listed as being between 0.1 and 0.2 meter; to be on the conservative side, one may take

$$\sigma_{altimetry} = 0.2\,m .$$

The variance from the last two sources of σ^2_{total} certainly contribute to its final value; if they are left out from these considerations the latter will be too optimistic, provided of course that the other variances are realistic. This has been examined upon considering

$$\sigma_{terrestrial\ parameters} = 0 , \qquad \sigma_{algorithm} = 0 .$$

The result obtained with all the above values is

$$\sigma_{total} = 5.15\,m .$$

However, the estimate of this quantity through the r.m.s. misclosure is

$$\hat{\sigma}_{total} = 3.66\,m .$$

This indicates that the theoretical value of 5.15 m is not overly optimistic, to the contrary, that it is too high when compared with the value obtained from the real data gathered over all the world's oceans with SEASAT.

The above result prompts the following comments. It is confirmed that the terrestrial parameters (the 14,14 subset of GEM 10 coefficients and the reference field parameters) present in the adjustment model are of excellent quality, and that assigning a zero sigma to this source of error is probably not far from the reality. A similar statement could be made with regard to the short-arc algorithm, in particular, with regard to the errors committed in the process of orbital integration with the given S.H. potential coefficients upon adhering to a seven-minute arc criterion. The high quality of SEASAT altimeter measurements and precise ephemeris is also confirmed. Finally, in considering that the real-data estimate of the total model sigma is appreciably lower than the theoretical value, the possibility exists that the covariance function used in computing the theoretical sigma due to the truncation may be too conservative, at least insofar as the geoid undulations for relatively low degree and order truncations are concerned.

APPENDIX 2

EFFICIENCY ANALYSIS FOR A REDUCTION IN THE NUMBER OF STATE VECTOR PARAMETERS

The observation equation system of satellite altimetry associated with an i-th arc can be written, in matrix notations, as

$$V_i = \ddot{A}_i\ddot{X}_i + \dot{A}_i\dot{X} + L_i \ , \tag{A2.1}$$

where

V_i = vector of n residuals on the i-th arc (containing n observations),

$\dot{X}$ = vector of corrections to m terrestrial parameters (here a truncated set of spherical-harmonic, S.H., potential coefficients),

$\ddot{X}_i$ = vector of corrections to 6 state vector (s.v.) parameters on the i-th arc,

$\dot{A}_i$ = $n \times m$ design matrix associated with $\dot{X}$,

$\ddot{A}_i$ = $n \times 6$ design matrix associated with $\ddot{X}_i$,

L_i = vector of n constant terms.

A more explicit form of the arrays in (A2.1) can be found, for example, in Chapter 2 of [Blaha, 1977], equation (2.22) in particular.

The starting point in the analysis can be identified with the results listed in Section 2.5 of [Blaha, 1975]. The (diagonal) weight matrix associated with the n observations is denoted P_i and the weight matrices associated with $\dot{X}$, $\ddot{X}_i$ are denoted $\dot{P}$, $\ddot{P}_i$, respectively. The total number of arcs is s, and there is no "terrestrial source" such as gravity anomalies present. The most important formulas in the current task are

$$\underset{(6,m)}{Q_i} = (\ddot{N}_i + \ddot{P}_i)^{-1}\,\bar{N}_i^T \,, \tag{A2.2a}$$

$$\underset{(m,m)}{\dot{\Lambda}} = [\sum_{i=1}^{s} (\dot{N}_i - \bar{N}_i Q_i) + \dot{P}]^{-1} ; \tag{A2.2b}$$

$$\underset{(m,1)}{\dot{X}} = \dot{\Lambda} \sum_{i=1}^{s} (\dot{C}_i - Q_i^T\,\ddot{C}_i) \,, \tag{A2.3a}$$

$$\underset{(6,1)}{\ddot{X}_i} = (\ddot{N}_i + \ddot{P}_i)^{-1}\,\ddot{C}_i - Q_i \dot{X} \,; \tag{A2.3b}$$

$$\underset{(m,m)}{\Sigma_{\dot{X}}} = \dot{\Lambda} \,, \tag{A2.4a}$$

$$\underset{(6,6)}{\Sigma_{\ddot{X}_i}} = (\ddot{N}_i + \ddot{P}_i)^{-1} + Q_i\,\dot{\Lambda}\,Q_i^T \,, \tag{A2.4b}$$

where

$$\underset{(m,m)}{\dot{N}_i} = \dot{A}_i^T\,P_i\,\dot{A}_i \,,$$

$$\underset{(m,6)}{\bar{N}_i} = \dot{A}_i^T\,P_i\,\ddot{A}_i \,,$$

$$\underset{(6,6)}{\ddot{N}_i} = \ddot{A}_i^T\,P_i\,\ddot{A}_i \,;$$

$$\underset{(m,1)}{\dot{C}_i} = \dot{A}_i^T\,P_i(-L_i) \,,$$

$$\underset{(6,1)}{\ddot{C}_i} = \ddot{A}_i^T\,P_i(-L_i) \,.$$

The corrections $\ddot{X}_i$ refer to the earth-fixed (E.F.) coordinate system. If $\ddot{Y}_i$ denotes the same corrections in the "i,c,o" system (in-track, crosstrack, orthogonal) discussed in Chapter 3 of [Blaha, 1977], then, in agreement with (3.22) and (3.23) of this reference, one can write

$$\ddot{Y}_i = \bar{R} \ddot{X}_i , \tag{A2.5a}$$

$$\ddot{X}_i = \bar{R}^T \ddot{Y}_i ; \tag{A2.5b}$$

the orthonormal matrix $\bar{R}$ is given as

$$\bar{R} = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} , \tag{A2.5c}$$

with the orthonormal matrix R of dimensions (3,3) expressed in (3.21) of the same reference. Since $\bar{R}^T \bar{R} = I$, equation (A2.1) can be rewritten as

$$V_i = (\ddot{A}_i \bar{R}^T)\ddot{Y}_i + \dot{A}_i\dot{X} + L_i ,$$

as if the first design matrix were $\ddot{A}_i \bar{R}^T$ and the orbital parameters were $\ddot{Y}_i$. According to (A2.5a), the corresponding weight matrix for $\ddot{Y}_i$, denoted $\ddot{\ddot{P}}_i$, is

$$\ddot{\ddot{P}}_i = \bar{R} \ddot{P}_i \bar{R}^T . \tag{A2.5d}$$

In fact, it is this (diagonal) matrix which is given a-priori and from which $\ddot{P}_i$ in the customary E.F. system of adjustment is derived as $\bar{R}^T \ddot{\ddot{P}}_i \bar{R}$.

With the above parameter transformation, (A2.2a) can be expressed in terms of the new parameters, $\ddot{Y}_i$, as

$$\tilde{Q}_i = (\bar{R}\,\ddot{N}_i\,\bar{R}^T + \ddot{\bar{P}}_i)^{-1}\,\bar{R}\,\bar{N}_i^T , \qquad \text{(A2.6a)}$$

which, due to (A2.5c), is $\bar{R}(\ddot{N}_i + \ddot{P}_i)^{-1}\,\bar{N}_i^T$. Similarly, parallel to (A2.2b) one has

$$\dot{\Lambda} = [\sum_{i=1}^{s} (\dot{N}_i - \bar{N}_i\,\bar{R}^T\,\tilde{Q}_i) + \dot{P}]^{-1} ; \qquad \text{(A2.6b)}$$

this expression is invariable with respect to the transformation between $\ddot{X}_i$ and $\ddot{Y}_i$, and remains identical to (A2.2b) as it should. This is confirmed immediately upon realizing, from (A2.6a), that

$$\bar{R}^T\,\tilde{Q}_i = Q_i .$$

Equations (A2.3) with $\ddot{Y}_i$ as orbital parameters (and thus $\ddot{A}_i\,\bar{R}^T$ as the first design matrix) become

$$\dot{X} = \dot{\Lambda} \sum_{i=1}^{s} (\dot{C}_i - \tilde{Q}_i^T\,\bar{R}\,\ddot{C}_i) , \qquad \text{(A2.7a)}$$

$$\ddot{Y}_i = (\bar{R}\,\ddot{N}_i\,\bar{R}^T + \ddot{\bar{P}}_i)^{-1}\,\bar{R}\,\ddot{C}_i - \tilde{Q}_i\,\dot{X} . \qquad \text{(A2.7b)}$$

For the same reason as above, $\dot{X}$ in (A2.7a) is seen to be invariable with regard to the s.v. parameter transformation and is identical to its form in (A2.3a). It can be further verified that upon pre-multiplying (A2.7b) by $\bar{R}^T$, (A2.3b) is recovered. Finally, equations (A2.4) similarly become

$$\Sigma_{\dot{X}} = \dot{\Lambda} , \tag{A2.8a}$$

$$\Sigma_{\ddot{Y}_i} = (\bar{R}\, \ddot{N}_i\, \bar{R}^T + \ddot{\bar{P}}_i)^{-1} + \tilde{Q}_i\, \dot{\Lambda}\, \tilde{Q}_i^T ; \tag{A2.8b}$$

if (A2.8b) is pre-multiplied by $\bar{R}^T$ and post-multiplied by $\bar{R}$, (A2.4b) is recovered.

Suppose next that the first two coordinate corrections in $\ddot{Y}_i$ are identically zero. This means that the in-track and crosstrack components of the s.v. are held fixed. If the satellite orbit is approximately circular, one can then say that, equivalently, the horizontal components of the s.v. are held fixed and only the vertical component, as well as the three velocity components, are subject to adjustment. Since differential changes along horizontal directions do not have any bearing on the satellite altimetry adjustment, such a simplification could be used as a legitimate means for reducing the computer run-time requirements. This is indeed the main motivation for the present analysis.

An efficient way to prevent the first two elements of $\ddot{Y}_i$ from entering the adjustment is to simply suppress them and be in the presence of a new vector $\ddot{Y}_i$ with only four elements. The matrix $\bar{R}^T$ in (A2.5b) is effectively reduced in size to four columns, the first two columns being likewise suppressed. ne can then proceed according to (A2.6), (A2.7) and (A2.8), keeping in mind that the dimensions are changed as follows: $\ddot{Y}_i \rightarrow (4,1)$, $\bar{R} \rightarrow (4,6)$, $\bar{R}^T \rightarrow (6,4)$, $\ddot{\bar{P}} \rightarrow (4,4)$, $\tilde{Q}_i \rightarrow (4,m)$, $\Sigma_{\ddot{Y}_i} \rightarrow (4,4)$. After the adjustment process is terminated, the s.v. parameters in the E.F. system and their variance-covariance matrix are obtained as

$$\underset{(6,1)}{\ddot{X}_i} = \underset{(6,4)}{\bar{R}^T} \; \underset{(4,1)}{\ddot{Y}_i} \, , \tag{A2.9a}$$

$$\underset{(6,6)}{\Sigma_{\ddot{X}_i}} = \underset{(6,4)}{\bar{R}^T} \; \underset{(4,4)}{\Sigma_{\ddot{Y}_i}} \; \underset{(4,6)}{\bar{R}} \, . \tag{A2.9b}$$

In the following, two approaches will be compared in efficiency: the original approach with no reduction in the number of parameters, equations (A2.2)-(A2.4), and the approach being analyzed, where the number of s.v. parameters is reduced from six to four, equations (A2.6)-(A2.9) with the dimensions changed accordingly. The efficiency is compared only with regard to the number of scalar multiplications in the adjustment algorithm. This represents, of course, only a small part of the total computer run-time; the formation of the observation equations, the input-output operations, etc., are not considered in this analysis. Accordingly, the actual savings achievable with the reduced number of s.v. parameters are likely to be much lower than those indicated by the present count. Although the number of scalar multiplications needed for an inversion of a (P,P) matrix is kP^3, where the coefficient of proportion (k) depends on the method used, P^3 (thus k=1) will be used here for the sake of simplicity. Since the weight matrix P_i is usually given as $(1/\sigma^2)I$, it will be assumed that the observation equations have been "normalized" (i.e., divided by σ) and thus the operations involving P_i will be disregarded.

<u>Original algorithm</u>. As indicated in the statement that followed (A2.5d), the weight matrix $\ddot{P}_i$ must be computed from the diagonal matrix $\ddot{\ddot{P}}_i$ given a-priori. According to (3.26) of [Blaha, 1977], one can write

$$\ddot{P}_i = \begin{bmatrix} R^T d^{-1} R & 0 \\ 0 & R^T \dot{d}^{-1} R \end{bmatrix},$$

where d^{-1}, $\dot{d}^{-1}$ are diagonal matrices of dimensions (3,3) and where R was introduced following (A2.5c). In a general case of the matrix product AB, where the dimensions of A, B are (n,m), (m,p), respectively, the number of scalar multiplications (called operations) needed to compute one element of the resulting matrix is m; the whole process thus requires mnp operations. If A is a diagonal matrix of dimensions (m,m) this number is reduced to mp (i.e., the number of elements in B). Thus $d^{-1}R$ above involves 9 operations. If A and B have dimensions (n,m) and (m,n), respectively, and their product is symmetric, the number of required operations is reduced from mn^2 to $m \cdot \frac{1}{2}n(n+1)$. Accordingly, the product $R^T(d^{-1}R)$ introduces another 18 operations. The total of operations needed for $\ddot{P}_i$ is then $2 \cdot 27 = 54$.

In keeping in mind the "normalization" property and the symmetry of certain matrices, the operations needed to form the basic adjustment building blocks are symbolically expressed as

$\dot{N}_i$ ... $\frac{1}{2}nm(m+1)$,

$\overline{N}_i$ ... 6nm ,

$\ddot{N}_i$ ... 21n ;

$\dot{C}_i$ ... nm ,

$\ddot{C}_i$ ... 6n .

In forming Q_i from (A2.2a), the matrix inversion accounts for $6^3=216$ operations and the multiplication of the inverted matrix by $\overline{N}_i^T$ accounts for 36m operations, in addition to forming $\ddot{P}_i$, $\ddot{N}_i$ and $\overline{N}_i$. One then has

$$\text{s matrices } Q_i \ \ldots \ s(21n + 54 + 216 + 6nm + 36m) \ . \qquad \text{(A2.10)}$$

Additional operations needed in computing $\dot{\Lambda}$ are those associated with the formation of $\dot{N}_i$ and with the computation of the symmetric matrix $\overline{N}_i\,Q_i$, the latter necessitating $3m(m+1)$ operations, for all s arcs; the indicated matrix inversion will then add m^3 operations, for the total of

$$\dot{\Lambda} \ \ldots \ s\left[\tfrac{1}{2}nm(m+1) + 3m(m+1)\right] + m^3 \ . \qquad \text{(A2.11)}$$

Except for the pre-multiplication by $\dot{\Lambda}$, the computation of $\dot{X}$ involves $s(nm + 6n + 6m)$ operations so that the total corresponds to

$$\dot{X} \ \ldots \ s(nm + 6n + 6m) + m^2 \ . \qquad \text{(A2.12)}$$

The formation of $Q_i\dot{X}$ requires 6m operations, which leads to

$$\text{s vectors } \ddot{X}_i \ \ldots \ s(36 + 6m) \ . \qquad \text{(A2.13)}$$

Finally, the product $\dot{\Lambda}\,Q_i^T$ accounts for $6m^2$ operations and the formation of a symmetric matrix $Q_i(\dot{\Lambda}\,Q_i^T)$ requires 21m operations; one thus has

$$\text{s matrices } \Sigma_{\ddot{X}_i} \ \ldots \ s(6m^2 + 21m) \ . \qquad \text{(A2.14)}$$

Upon summing up the number of the operations in (A2.10)-(A2.14), one can write

$$\text{total} \ldots s[306+27n+7nm+69m+\tfrac{1}{2}nm(m+1)+3m(m+1)+6m^2] + m^2+m^3 \ . \qquad \text{(A2.15)}$$

The number of S.H. potential coefficients considered in this study corresponds to a (14,14) truncated model, or

$$m = (14+1)^2 = 225 \ . \qquad \text{(A2.16)}$$

With this number, (A2.15) becomes

$$\text{total} \ldots s(472{,}131 + 27{,}027n) + 11{,}441{,}250. \qquad \text{(A2.17)}$$

Algorithm with reduced number of s.v. parameters. In analyzing this second algorithm, one works with equations (A2.6)-(A2.9), upon taking into account the reduction in dimensions indicated prior to (A2.9a). The product $\bar{R}\ddot{N}_i\bar{R}^T$ requires the following number of operations: 21n (due to the formation of $\ddot{N}_i$), plus $6 \cdot 6 \cdot 4 = 144$ (due to the product $\ddot{N}_i\bar{R}^T$), plus $6 \cdot \tfrac{1}{2} \cdot 4 \cdot 5 = 60$ (due to the remaining product yielding a symmetric matrix), for a total of $21n + 204$ operations. However, this product could be formed differently, as $(\bar{R}\ddot{A}_i^T)(\ddot{A}_i\bar{R}^T)$, where the "normalization" is understood. The product inside the first parentheses requires 24n operations and the final product yielding a symmetric matrix requires $n \cdot \tfrac{1}{2} \cdot 4 \cdot 5 = 10n$ operations, for a total of 34n operations. Although at first sight this approach seems to be less advantageous than the one with $21n + 204$ operations described above, the subsequent formation of $(\bar{R}\ddot{A}_i^T)\dot{A}_i$ will necessitate only 4nm additional operations, as opposed to $6nm + 24m$ operations introduced via the product $\overline{R}\,\overline{N}_i^T$. Accordingly, the most economical procedure in forming all the matrices $\tilde{Q}_i$ can be symbolized by

$$\text{s matrices } \tilde{Q}_i \;\ldots\; s(34n + 4nm + 64 + 16m) \;, \qquad \text{(A2.18)}$$

where the 64 operations stand for the inversion of a (4,4) matrix and the 16m operations represent the final multiplication between the inverted matrix and the matrix $\overline{R}\,\overline{N}_i^T$ of dimensions (4,m).

With regard to (A2.6b), the formation of $\dot{N}_i$ was already seen to require $\frac{1}{2}nm(m+1)$ operations and the product $(\overline{N}_i\,\overline{R}^T)\tilde{Q}_i$, resulting in a symmetric matrix, requires 2m(m+1) operations; this is repeated for all the arcs. If m^3 is added to this number due to the indicated matrix inversion, one obtains

$$\dot{\Lambda} \;\ldots\; s\,\left[\tfrac{1}{2}nm(m+1) + 2m(m+1)\right] + m^3 \;. \qquad \text{(A2.19)}$$

The product $(\overline{R}\,\ddot{A}_i^T)L_i$, equivalent to $\overline{R}\,\ddot{C}_i$, accounts for 4n operations and the product $\tilde{Q}_i^T(\overline{R}\,\ddot{C}_i)$ necessitates 4m operations. If the formation of $\dot{C}_i$ (nm operations) is also considered, if the whole process is repeated for every arc, and if the pre-multiplication of $\dot{\Lambda}$ (m^2 operations) takes place according to (A2.7a), one obtains

$$\dot{X} \;\ldots\; s(4n + 4m + nm) + m^2 \;. \qquad \text{(A2.20)}$$

When $\ddot{Y}_i$ is considered, the product of a matrix of dimensions (4,4), already known, and a vector of dimensions (4,1), also known, represents 16 operations; the product $\tilde{Q}_i\,\dot{X}$ necessitates additional 4m operations, yielding

$$s \text{ vectors } \ddot{Y}_i \ \ldots \ s(16+4m) \ . \qquad \text{(A2.21)}$$

The formation $\tilde{Q}_i(\dot{\Lambda}\tilde{Q}_i^T)$ necessitates $4m^2$ operation plus 10m operations (the final matrix is symmetric), so that

$$s \text{ matrices } \Sigma_{\ddot{Y}_i} \ \ldots \ s(4m^2+10m) \ . \qquad \text{(A2.22)}$$

Due to (A2.9), the following is added to the above sequence:

$$s \text{ vectors } \ddot{X}_i \ \ldots \ 24s \ , \qquad \text{(A2.23)}$$

$$s \text{ matrices } \Sigma_{\ddot{X}_i} \ \ldots \ 180s \ ; \qquad \text{(A2.24)}$$

the number of operations in these last two equations is very small compared to that in any of the tasks carried out previously.

All the operations in (A2.18)-(A2.24) are now added, resulting in

$$\text{total} \ \ldots \ s[284+38n+5nm+34m + \tfrac{1}{2}nm(m+1)+2m(m+1)+4m^2]+m^2+m^3 \ . \qquad \text{(A2.25)}$$

With m given in (A2.16), this becomes

$$\text{total} \ \ldots \ s\ (312{,}134 + 26{,}588\,n) + 11{,}441{,}250 \ . \qquad \text{(A2.26)}$$

<u>Comparison of the two algorithms</u>. Upon comparing the results (A2.10) with (A2.18), etc., a reduction in the number of operations in the second algorithm can be noticed at every step. However, by far the largest number of operations is due to the formation of $\dot{N}_i$ on each arc, resulting in $\frac{1}{2}snm(m+1)$ operations for all s arcs. This computer burden is identical for both algorithms, as may be gathered upon comparing (A2.11) and (A2.19). All the other computer savings materialized in the second algorithm are largely overshadowed by this fact.

As a matter of interest, Table A2.1, constructed with the aid of (A2.17) and (A2.26), shows the savings materialized with the second algorithm for a few selected values of n and s. It is apparent that the savings are not impressive, especially if one keeps in mind that the savings listed are only a fraction of the final computer savings. Furthermore, the second algorithm impairs the rigor of the solution for non-circular orbits, in the sense that the larger orbit's eccentricity is, the larger errors introduced into the model with the reduced number of s.v. parameters are. Since this price is too heavy considering the insignificant run-time savings obtainable with the second algorithm, it is recommended that the original (first) algorithm featuring six s.v. parameters per arc in the short-arc mode of satellite altimetry be retained.

n = 100			
s	NO. OF OPERATIONS IN 1ST ALGORITHM	NO. OF OPERATIONS IN 2ND ALGORITHM	ECONOMY REALIZED IN 2ND ALGORITHM
100	328,924,350	308,534,650	6.20%
1,000	3,186,272,200	2,982,375,250	6.40%
large	...	...	6.42%
n = 1000			
100	2,761,354,350	2,701,454,650	2.17%
large	...	...	2.18%

Table A2.1

Comparison of two adjustment algorithms in the short-arc mode of satellite altimetry

APPENDIX 3
ALGORITHM FOR ARTIFICIAL LOWERING OF STATE VECTOR SIGMAS

The algorithm which implements the artificial weight changes for the state vector parameters as described in Chapter 2 will be now presented in detail. It is based on the results of Chapter 2 of [Blaha, 1975]. As on page 15 of this reference, a set of observation equations for the i-th arc is symbolized by

$$V_i = [\dot{A}_i \,\vdots\, \dots\, \ddot{A}_i \,\dots\,] \begin{bmatrix} \dot{X}_i \\ \hdashline \vdots \\ \ddot{X}_i \\ \vdots \end{bmatrix} + L_i \ , \qquad \text{(A3.1)}$$

where $\dot{X}$ represents the corrections to the S.H. potential coefficients, $\ddot{X}_i$ represents the corrections to the six s.v. parameters on the i-th arc, $\dot{A}_i$ and $\ddot{A}_i$ denote the pertinent matrices of partial derivatives, and where

$$L_i = L_i^o - L_i^b \ ,$$

L_i^o symbolizing the computed values of the observables and L_i^b symbolizing the measured values of the same quantities (here the altimeter observations). Every arc must be weighted independently, otherwise the short-arc algorithm breaks down. In the actual computations not only the groups of observations on separate arcs but also the individual observations are usually weighted independently; in fact, the latter are usually attributed equal weights. In any event, the weight matrix for all the observations on the i-th arc is denoted as P_i, the (original) weight matrix associated with the s.v. parameters on this arc is denoted as $\ddot{P}_i$, and the weight matrix associated with the terrestrial parameters (S.H. potential coefficients) is denoted as $\dot{P}$.

The formulas in [Blaha, 1975] have been adapted to the case at hand along the following lines: all the observations are considered to come from a satellite source (here the SEASAT altimeter), whereas the terrestrial and other information is contained in the weighted S.H. coefficients; the parameters are weighted at their approximate (input) values; and the adjustment process does not proceed by iterations (otherwise already in the second iteration the parameters would be weighted at other than the approximate values which would be updated from the first iteration). In analogy to Appendix 2, the final formulas involving a total of s satellite arcs thus read:

$$\dot{N}_i = \dot{A}_i^T P_i \dot{A}_i \,, \quad \overline{N}_i = \dot{A}_i^T P_i \ddot{A}_i \,, \quad \ddot{N}_i = \ddot{A}_i^T P_i \ddot{A}_i \,,$$
$$\dot{C}_i = \dot{A}_i^T P_i (-L_i) \,, \quad \ddot{C}_i = \ddot{A}_i^T P_i (-L_i) \;;$$

$$Q_i = (\ddot{N}_i + \ddot{P}_i)^{-1} \overline{N}_i^T \;; \tag{A3.2}$$

$$\ddot{X}_i = (\ddot{N}_i + \ddot{P}_i)^{-1} \ddot{C}_i - Q_i \dot{X} \,, \tag{A3.3a}$$
$$\Sigma_{\ddot{X}_i} = (\ddot{N}_i + \ddot{P}_i)^{-1} + Q_i \dot{\Lambda} Q_i^T \;; \tag{A3.3b}$$

$$\dot{\Lambda} = \Big[\sum_{i=1}^{s} (\dot{N}_i - \overline{N}_i Q_i) + \dot{P}\Big]^{-1} \;;$$

$$\dot{X} = \dot{\Lambda} \sum_{i=1}^{s} (\dot{C}_i - Q_i^T \ddot{C}_i) ,$$

$$\Sigma_{\dot{X}} = \dot{\Lambda} .$$

The only formulas in which the weights should artificially change (from $\ddot{P}_i$ to $\ddot{P}_i'$) are (A3.3a) and (A3.2), the latter strictly in conjunction with the former. This simple statement can be formally demonstrated as follows. Since the values $\dot{X}$ are not to be affected, one can proceed as if adjusting only $\ddot{X}_i$ associated with the new weight matrix $\ddot{P}_i'$. The set of observation equations (A3.1) thus becomes:

$$V_i = [\ldots \ddot{A}_i \ldots] \begin{bmatrix} \vdots \\ \ddot{X}_i \\ \vdots \end{bmatrix} + (L_i + \dot{A}_i \dot{X}) ,$$

where $\dot{X}$ is now a part of the constant terms. Upon applying the least-squares criterion to each such set independently -- this is possible since the sets of state vector parameters are weighted independently for each arc and the same holds for the observations -- the new normal equations (as yet unweighted) are

$$\ddot{A}_i^T P_i \ddot{A}_i \ddot{X}_i = \ddot{A}_i^T P_i (-L_i - \dot{A}_i \dot{X})$$

or

$$\ddot{N}_i \ddot{X}_i = \ddot{C}_i - \overline{N}_i^T \dot{X} ,$$

from which the solution for $\ddot{X}_i$ follows as

$$\ddot{X}_i = (\ddot{N}_i + \ddot{P}_i')^{-1}\, \ddot{C}_i - Q_i'\, \dot{X}\ , \tag{A3.4a}$$

$$Q_i' = (\ddot{N}_i + \ddot{P}_i')^{-1}\, \overline{N}_i^T\ . \tag{A3.4b}$$

Except for $\ddot{P}_i'$, the last two expressions have the same form as their counterparts in (A3.3a) and (A3.2), respectively.

In the original algorithm the matrices $(\ddot{N}_i + \ddot{P}_i)^{-1}$ and Q_i are saved for each arc (as is the vector $\ddot{C}_i$). Since (A3.3b), etc., are not subject to the artificial weight changes, these two matrices will remain unchanged and both $(\ddot{N}_i + \ddot{P}_i')^{-1}$ and Q_i' appearing in equations (A3.4) will be generated from them. In this process, the approximation

$$(\ddot{N}_i + \ddot{P}_i')^{-1} \simeq (\ddot{N}_i + \ddot{P}_i + \ddot{P}_i')^{-1} \tag{A3.5}$$

can be used because the diagonal elements in $\ddot{P}_i$ will be many times, perhaps hundred-fold, smaller than those in $\ddot{P}_i'$; accordingly, the "overweighting" by $\ddot{P}_i$ is of no concern in this process which in itself is only approximate. (The removal of $\ddot{P}_i$ would be unnecessarily time-consuming; the sigmas of the state vectors were originally given for the in-track, crosstrack and approximately "up" directions, from which the $\ddot{P}_i$ in the earth-fixed, E.F., coordinate system of adjustment was computed.)

The advantage of heavily weighting all three positional components of the state vectors is now apparent. If only the radial component were so weighted -- which would be all the present artificiality would require -- the (small) sigma of this component would have to be transformed from the "track" to the E.F. coordinate system of adjustment. On the other hand, if all three positional sigmas are equal they give rise to an "error sphere" which remains a sphere in any coordinate system. Accordingly, if the state vector is attributed the positional sigmas of 0.16 m, 0.16 m, 0.16 m, then $\ddot{P}_i'$ follows immediately as a matrix whose first three diagonal elements are $1/(0.16\,m)^2$ and all the other elements are zero. In this particular case the weight of the radial component is being artificially increased hundred-fold (the original sigma was given as 1.6 m).

As a result of this development the new matrices needed in producing the artificial weight changes are generated by the following algorithm:

$$(\ddot{N}_i + \ddot{P}_i')^{-1} \simeq (G_i + \ddot{P}_i')^{-1} \ , \qquad \text{(A3.6)}$$

$$Q_i' = (\ddot{N}_i + \ddot{P}_i')^{-1} \, G_i \, Q_i \ , \qquad \text{(A3.7)}$$

where G_i is computed as

$$G_i = [(\ddot{N}_i + \ddot{P}_i)^{-1}]^{-1} \ . \qquad \text{(A3.8)}$$

APPENDIX 4

SEA SURFACE EFFECTS NOT INCLUDED IN SEASAT ALTIMETRY MODEL

The sea surface effects which are not to be included in SEASAT altimetry model will now be briefly described. The description proceeds according to the order in which these effects were listed in Chapter 3, where they were also classified in seven basic categories. Their numbering and classification need not be repeated here.

Variations of the speed of the earth's rotation. This problem area is concerned with the mean sea level (MSL) variations which are very small, on the order of 0.5 cm at the most. The variation in the earth's rotation, if needed, would be computed from the variation in the length of days. Due to the amplitude of this contribution being an order of magnitude smaller than that of the other sources treated in the present study, it is left out of consideration.

Wind effects. The most important direct effect of winds on the sea surface level is the "piling up" of water in one area with the corresponding depression in another area. In more pronounced cases this phenomenon is known under the name "storm surge". However, these effects are usually of short duration and depend on local conditions; in particular, they are often associated with an atmospheric depression passing over an area. Even if local phenomena were of interest, meaningful modeling of such effects would be exceedingly difficult because of their complexity and irregularity. The present study is concerned with sea level variations and geoidal modeling in large oceanic area covered by SEASAT altimetry.

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SEASAT ALTIMETRY ADJUSTMENT MODEL INCLUDING TIDAL AND OTHER SEA--ETC(U)
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Such areas are affected the least by winds whose influence is mostly felt along the coasts (against which the water piles up), in shallow water, estuaries, land-locked basins, or adjacent and Mediterranian-type seas. The local characteristics of these effects constitute the main reason for not including the storm surges among the variations whose contribution can be countered by an appropriate correction to altimeter measurements. Instead, these variations must be considered as simply contributing to the "noise" of the system.

Evaporation and precipitation. These sources of sea level changes are of little significance (especially their direct contribution to water level in open ocean) and are therefore left out of consideration.

Currents. These effects are too complex to be included, on a large scale, in the computation of an appropriate correction to any meaningful accuracy. They are interrelated with the effects of atmospheric pressure, wind, Coriolis force (generated by the earth's rotation) and other factors. The changes in sea level due directly to currents are relatively small and localized, and their effect on SEASAT altimetry could likely be omitted even if the computation of a correction for that purpose were feasible.

Vertical movement of the earth's crust. These changes are very slow, resulting in sea level changes of no more than a few mm per year. They would be of interest for geoidal comparisons several decades apart. However, this is not related to the present study, concerned with an improved determination of the oceanic geoid with the aid of SEASAT altimetry rather than with long-term monitoring of geoidal changes.

Melting or forming of continental ice, etc. The contribution of these effects is even smaller than that of the preceding item (the sea level is affected hardly more than 1 mm per year) and is likewise left out of this study.

Coastal and other local phenomena. The SEASAT ground tracks cover the globe's ocean surface (except for the polar regions) in an approximate $1^0 \times 1^0$ grid of ascending and descending passes. Although a number of altimeter observations exist between the intersections on any arc, the finest uniform resolution of the geoid surface one can expect to achieve is somewhat coarser than a 1^0 resolution (i.e., the shortest half-wavelength representing the geoidal detail in any oceanic area is somewhat longer than 1^0). A resolution which could be regarded with confidence because of a reasonable amount of data filtering through an adjustment process would certainly be a 2^0 resolution, although for some purposes or in specific areas a finer resolution might be desirable. Be that as it may, it is clear that small water basins of any kind, isolated from a continuous water surface where a number of SEASAT crossings exist, would be of little value to the global SEASAT altimetry adjustment. In fact, considering that all the arcs shorter than 3^0 have been eliminated from the adjustment at a pre-processing stage, small bodies of water would contain virtually no altimeter observations (a long and narrow body of water could contain ground tracks if it is oriented along a pass, but only in that direction without any intersections).

Even if the orbital parameters could be considered perfect, which would reduce the need for intersecting arcs, the coastal waters as well as various water basins and shallow water areas would present problems of a different kind. These would be the areas where the irregular, complex and unpredictable effects due to the atmospheric pressure, winds, currents, etc., would most degrade the altimeter measurements (here these effects would be much more pronounced than in open ocean, and would be much more difficult, if not impossible, to model or correct for). For example, the tidal changes in sea basins, including the Mediterranian-type seas, are very small, often reaching only a few centimeters, and are secondary to various irregular changes such as those generated by the wind action described earlier. Another complication which would arise in partly or totally enclosed water basins would be due to seiches, or standing-wave oscillations.

Either of the two main reasons just described -- i.e., pertaining to 1) length of SEASAT passes, and 2) sea level changes in small water basins, etc., impossible to model with a meaningful accuracy -- tends to justify adjusting SEASAT altimeter data gathered over open ocean only. The measurements gathered over water basins, if present, are not envisioned to be eliminated from the adjustment process, but it should be borne in mind that they are of a lower quality. In terms of the present SEASAT altimetry model, the coastal and other local phenomena (in partly or totally enclosed water basins, shallow water, adjacent and Mediterranian-type seas, estuaries, etc.) are to be eliminated from any further consideration.

Other phenomena. A typical example of a phenomenon in this category is a tsunami, or seismic sea wave. It is a long surface wave caused by an earthquake or other underwater erruption and it clearly constitutes an isolated event. It is therefore left out of consideration.

APPENDIX 5

REFINEMENT OF THE FORMULA GIVING THE AVERAGE EQUILIBRIUM TIDE

The tidal undulation (N) for the equilibrium tide was treated in the first part of [Blaha, 1980]; this reference is henceforth abbreviated as [B]. Initially, only the moon's tide-rising effect was considered and the average earth-moon distance was adopted for this purpose. The sun's effect was then considered along similar lines. Next, the tidal undulation was averaged in time for a given point by averaging it for the moon, for the sun, and adding algebraically the two effects. The resulting undulation was expressed as a function of the geocentric latitude (ϕ) as follows:

$$\overline{N} = 0.148\,\text{m}\,(\cos 2\phi - 1/3) \; ; \qquad \text{(A5.1)}$$

the deviation from the formula (5.12) in [B] consists in writing the result with three significant digits and in employing the overbar to denote the time average. The above formula was obtained, however, through neglecting the inclination of the moon's orbit in order to achieve simplifications in the derivation. A rudimentary consideration indicated that the error thus committed would not surpass 1.9 cm in the worst case. The present analysis aims at developing a more exact formula than (A5.1) and, at the same time, at showing that the error committed in (A5.1) is, in fact, an order of magnitude smaller than previously indicated.

The starting point in this demonstration is equation (5.1) of [B] giving the tidal undulation at a point as a function of the zenith distance (theoretically reduced to the center of the earth) to the celestial body in question:

$$N = k(\cos 2z + 1/3) , \qquad (A5.2)$$

where z is the zenith distance and k becomes either k_1 (for the moon) or k_2 (for the sun) as in (5.2) of [B] which had been adapted from [Bomford, 1975], page 272:

$$k_1 = 0.267\,m , \qquad (A5.3a)$$

$$k_2 = 0.123\,m . \qquad (A5.3b)$$

If both the moon's and sun's effects are to be evaluated at conjunction or opposition, it follows that the value to be used is

$$k_1 + k_2 = 0.390\,m . \qquad (A5.3c)$$

The numerical value of k_1 is obtained, in accordance with [Bomford, 1975], as

$$k_1 = V_M / \bar{g} , \qquad (A5.4a)$$

$$V_M = (3/4)GMa^2/r_M^3 = (3/4)GEa^2(c/r_M^4)(1 - c/r_M)^{-1} , \qquad (A5.4b)$$

where $\bar{g}$ is the average terrestrial gravity, G is the gravitational constant, a is the earth's mean radius, c and r_M are the distances from the earth's center to the barycenter of the earth-moon system and to the moon's center, respectively; M and E indicate the moon's and the earth's masses, respectively.

In order to express z as a function of time, equations (3.1)-(3.3) of [Mueller, 1969] were used in (5.6) of [B] yielding (5.7) essentially as follows:

$$\cos 2z = -\sin^2\delta \cos 2\phi + \cos^2\delta \cos 2\phi \cos^2 h - \cos^2\delta \sin^2 h + \sin 2\delta \sin 2\phi \cos h, \tag{A5.5a}$$

where h and δ are the hour angle and declination of the celestial body, respectively. This expression can readily be developed into a more advantageous form. Upon combining the second and third terms on the right-hand side, $\cos^2\delta \cos^2\phi \cos 2h - \cos^2\delta \sin^2\phi$ is obtained; the latter term combined with the (original) first term yields $-\sin^2\delta - \sin^2\phi + 3 \sin^2\delta \cdot \sin^2\phi$, so that

$$\cos 2z = 3 \sin^2\delta \sin^2\phi - \sin^2\delta - \sin^2\phi + \sin 2\delta \sin 2\phi \cos h + \cos^2\delta \cos^2\phi \cos 2h . \tag{A5.5b}$$

From (A5.2) and (A5.5b) it follows that

$$N = k \left[(3 \sin^2\delta - 1)(\sin^2\phi - 1/3) - \sin 2\delta \sin 2\phi \cos h + \cos^2\delta \cos^2\phi \cos 2h\right] , \tag{A5.6}$$

which corresponds to equation (3) in [Lisitzin, 1974].

Next, formulas for the mean of a few trigonometric functions in the form needed in this study are developed. The mean over the interval

(τ_1, τ_2) is identified by an overbar. The result for $\overline{\cos 2\tau}$ where τ covers $18.6 \cdot 2\pi$ radians is now derived in detail for later use; in particular,

$$\overline{\cos 2\tau} = 1/(\tau_2-\tau_1)\int_{\tau_1}^{\tau_2} \cos 2\tau \, d\tau = \tfrac{1}{2}(\sin 2\tau_2 - \sin 2\tau_1)/(\tau_2-\tau_1) \ . \quad \text{(A5.7a)}$$

which also is

$$\overline{\cos 2\tau} = \cos(\tau_1+\tau_2)\,\sin(\tau_2-\tau_1)/(\tau_2-\tau_1) \ . \quad \text{(A5.7b)}$$

From either (A5.7a) or (A5.7b) it follows that $\overline{\cos 2\tau}$ approaches zero as the angle $\tau_2-\tau_1$ approaches $n\pi$, where n is an integer, or as this angle becomes large. In order to qualify the last statement, $\tau_2-\tau_1$ is taken as $18.6 \cdot 2\pi = 116.87$. The sine of this angle is -0.5878 while $\cos(\tau_1+\tau_2)$ can be taken as 1 or -1 in order to account for the worst possible situation. In the present case, one such choice of τ_1 and τ_2 (in degrees) is $\tau_1 = 72^0$, $\tau_2 = 288^0 + 18 \cdot 360^0$, so that

$$\tau_2-\tau_2 = 216^0 + 18 \cdot 360^0 \ , \qquad \tau_1+\tau_2 = 19 \cdot 360^0 \ ,$$

and

$$\overline{\cos 2\tau} = 0 - 0.0050 \ , \quad \text{(A5.8)}$$

where the second term on the right-hand side is the error in case $\overline{\cos 2\tau}$ is considered to be zero.

Listed below are the results for all the mean values needed; "interval" denotes the interval $\Delta\tau=\tau_2-\tau_1$ and "large", when referring to this interval, indicates that it is large enough so that the error is negligible.

$$\overline{\sin\tau} = -(\cos\tau_2 - \cos\tau_1)/\Delta\tau \ , \tag{A5.9a}$$

interval $n(2\pi)$ or large ... $\overline{\sin\tau} = 0$;

$$\overline{\cos\tau} = (\sin\tau_2 - \sin\tau_1)/\Delta\tau \ , \tag{A5.9b}$$

interval $n(2\pi)$ or large ... $\overline{\cos\tau} = 0$;

$$\overline{\sin 2\tau} = -\tfrac{1}{2}(\cos 2\tau_2 - \cos 2\tau_1)/\Delta\tau \ , \tag{A5.9c}$$

interval $n\pi$ or large ... $\overline{\sin 2\tau} = 0$;

$$\overline{\cos 2\tau} = \tfrac{1}{2}(\sin 2\tau_2 - \sin 2\tau_1)/\Delta\tau \ , \tag{A5.9d}$$

interval $n\pi$ or large ... $\overline{\cos 2\tau} = 0$,

interval $18.6 \cdot 2\pi$ ... max. error = 0.0050 ;

$$\overline{\sin^2\tau} = \tfrac{1}{2}(1-\overline{\cos 2\tau}) \ , \tag{A5.9e}$$

interval $n\pi$ or large ... $\overline{\sin^2\tau} = \tfrac{1}{2}$,

interval $18.6 \cdot 2\pi$ ... max. relative error = 0.5% ;

$$\overline{\cos^2\tau} = \tfrac{1}{2}(1+\overline{\cos 2\tau}) \ , \tag{A5.9f}$$

interval $n\pi$ or large ... $\overline{\cos^2\tau} = \tfrac{1}{2}$.

In view of the above relations, the formula (A5.6) simplifies when subjected to an averaging process over a long time interval, for example several years. Whether applied for the moon or for the sun, h covers the interval 2π thousands of times in this case. From (A5.6) one thus obtains

$$\overline{N} = k\ (3\ \overline{\sin^2\delta} - 1)(\sin^2\phi - 1/3) \equiv \tfrac{1}{2}k\ (\cos 2\phi - 1/3)(1 - 3\ \overline{\sin^2\delta})\ , \tag{A5.10}$$

where use has been made of (A5.9b) and (A5.9d) with h replacing τ.

Average effect of the moon. With the purpose of expressing $\sin^2\delta$, equation (3.11) of [Mueller, 1969] is utilized:

$$\sin\delta\ =\ \cos\beta\ \sin\lambda\ \sin\varepsilon + \sin\beta\ \cos\varepsilon\ ; \tag{A5.11}$$

the angles appearing on the right-hand side of this equation as well as other elements of the moon's orbit are shown in Figure A5.1. The quantity λ_N designates the (ecliptic) longitude of the ascending node of the moon's orbit whose period is approximately 18.6 years. From the figure it follows that

$$\lambda\ =\ \lambda_N + \lambda'\ . \tag{A5.12}$$

The period of κ is 27.3 days which means that the $\Delta\kappa$ interval can indeed be assumed to be large when averaging over several years, in particular, over the 18.6 years just mentioned. The motivation for adopting this averaging frame stems from the fact that "... the analysis of the more important tidal constituents in the oceans should cover a period corresponding to the revolution of the node of the lunar orbit, i.e., approximately 19 years" as stated on page 13 of [Lisitzin, 1974]. The numerical values which will be eventually used when evaluating the moon's effect are

$$\varepsilon \simeq 23.5^{0} \ , \tag{A5.13a}$$

$$i \simeq 5^{0} \ . \tag{A5.13b}$$

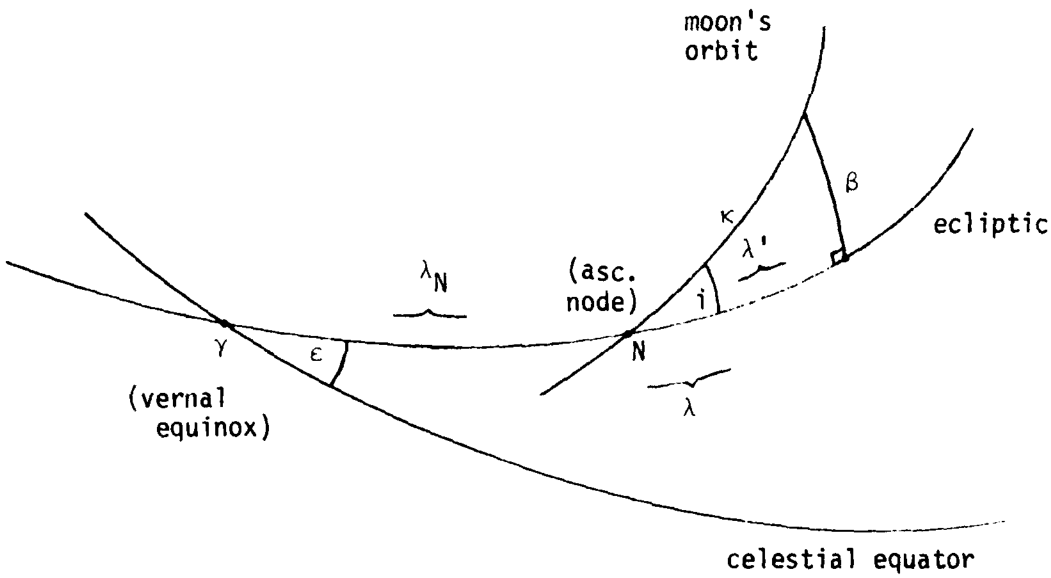

Figure A5.1
Elements of the moon's orbit

From the right spherical triangle of Figure A5.1 the following relationships are deduced:

$$\sin\beta = \sin i \sin\kappa , \tag{A5.14a}$$

$$\cos\beta \cos\lambda' = \cos\kappa , \tag{A5.14b}$$

$$\cos\beta \sin\lambda' = \cos i \sin\kappa . \tag{A5.14c}$$

If equations (A5.12) and (A5.14 a-c) are used in (A5.11) it follows that

$$\sin\delta = (\cos i \sin\kappa \cos\lambda_N - \cos\kappa \sin\lambda_N) \sin\varepsilon + \sin i \sin\kappa \cos\varepsilon . \tag{A5.15}$$

Upon squaring, regrouping the terms and using the double-angle sine formula, one has

$$\begin{aligned} \sin^2\delta = {} & \sin^2\varepsilon(\cos^2 i \sin^2\kappa \cos^2\lambda_N + \cos^2\kappa \sin^2\lambda_N) + \cos^2\varepsilon \sin^2 i \sin^2\kappa \\ & + \tfrac{1}{2} \sin^2\varepsilon \cos i \sin 2\kappa \sin 2\lambda_N \\ & + \tfrac{1}{2} \sin 2\varepsilon (\sin 2i \sin^2\kappa \cos\lambda_N + \sin i \sin 2\kappa \sin\lambda_N) . \end{aligned} \tag{A5.16}$$

By virtue of the formula (A5.9c) applied for κ in an interval of several years (not necessarily 18.6 years) the second line and the second term of the third line in (A5.16) would vanish. However, the first term of the third line would not vanish unless λ_N covers the interval of 18.6 years (or multiples thereof) as is seen from equation (A5.9b). But since such is the desired time interval, this term vanishes; in fact, due to λ_N alone both the second and third lines in (A5.16) vanish.

Upon applying (A5.9 e,f) to the first line of (A5.16) -- which amounts to replacing all of $\sin^2\kappa$, $\cos^2\kappa$, $\sin^2\lambda_N$, $\cos^2\lambda_N$ by ½ in the indicated averaging procedure -- one finally deduces

$$\overline{\sin^2\delta} = \tfrac{1}{2}\sin^2\varepsilon + \tfrac{1}{2}\sin^2 i\,[1-(3/2)\sin^2\varepsilon] \qquad \text{(A5.17a)}$$

and thus

$$1-3\,\overline{\sin^2\delta} = [1-(3/2)\sin^2\varepsilon]\,[1-(3/2)\sin^2 i]\ . \qquad \text{(A5.17b)}$$

The moon's effect is now designated by the index "1". Substituting (A5.17b) into (A5.10), one writes

$$\overline{N}_1 = \tfrac{1}{2}\,k_1\,(\cos2\phi - 1/3)[1-(3/2)\sin^2\varepsilon][1-(3/2)\sin^2 i]\ . \qquad \text{(A5.18)}$$

This expression will shortly be combined with a comparable relation giving the average tidal undulation due to the sun's effect, in order to produce the total average undulation due to both celestial bodies.

Average effect of the sun. The declination of the sun can be computed from (A5.11) with $\beta\equiv0$, in particular,

$$\sin\delta = \sin\lambda\,\sin\varepsilon\ ; \qquad \text{(A5.19)}$$

accordingly,

$$\overline{\sin^2\delta} = \overline{\sin^2\lambda}\,\sin^2\varepsilon\ . \qquad \text{(A5.20)}$$

Since the mean is sought for an interval of 18.6 years which corresponds to $18.6 * 2\pi$ in λ, in agreement with (A5.9e) one can take

$$\overline{\sin^2\lambda} = \tfrac{1}{2}\ , \text{ max. rel. error} = 0.5\%$$

and

$$\overline{\sin^2\delta} = \tfrac{1}{2}\sin^2\varepsilon, \text{ max. rel. error} = 0.5\%. \tag{A5.21}$$

The sun's effect is designated by the index "2" and (A5.21) is used in conjunction with (A5.10), giving

$$\overline{N}_2 = \tfrac{1}{2}\, k_2\, (\cos 2\phi - 1/3)[1 - (3/2)\sin^2\varepsilon]\ . \tag{A5.22}$$

The maximum relative error in this expression is 0.16%, due to the error in the last parenthesis evaluated through (A5.13a). But since the extreme value of (A5.22) reaches only -0.062 m (for the poles), its error can be safely ignored -- it could reach 0.1 mm in the worst possible case. The formula giving the average tidal undulation due to the sun's effect is thus (A5.22).

Combined average effect. The average tidal undulation due to both the moon's and sun's effects is obtained by adding algebraically the two individual tidal undulations as given by (A5.18) for the moon and by (A5.22) for the sun, namely

$$\overline{N} = \overline{N}_1 + \overline{N}_2\ , \tag{A5.23}$$

where $\overline{N}$ is the final combined value. The values needed in expressing (A5.23) are

$$k \;=\; k_1 + k_2 = 0.390\,\mathrm{m}\,, \qquad \text{(A5.24a)}$$

$$k_1 = 0.685\,k\,, \qquad \text{(A5.24b)}$$

which follows from equations (A5.3). Equations (A5.18, 22, 23, 24) thus yield

$$\overline{N} = \tfrac{1}{2}k\,(1 - 1.03\,\sin^2 i)(\cos 2\phi - 1/3)\left[1 - (3/2)\,\sin^2\varepsilon\right]\,, \qquad \text{(A5.25)}$$

$(3/2)k_1$ having been replaced by $1.03\,k$, as per (A5.24b).

The first parenthesis on the right-hand side of (A5.25) yields 0.99218 upon using (A5.13b) and the last parenthesis yields 0.7615 upon using (A5.13a). With k given in (A5.24a), the final result is

$$\overline{N} \;=\; 0.147\,\mathrm{m}\,(\cos 2\phi \;-\; 1/3)\;. \qquad \text{(A5.26)}$$

It is to be noted that if the correction term $1.03\,\sin^2 i$ in (A5.25) is neglected -- and thus the moon's orbit is assumed in the plane of the ecliptic as was done in [B] -- a relative error of only 0.8% is committed (i.e., the difference between unity and the value 0.99218 above). This error can safely be neglected and the formulas (5.9) and (5.12) of [B] can be regarded as reasonably accurate. In any case, the corresponding, more rigorous formulas are now (A5.25) and (A5.26), respectively. When (A5.26) is compared with (A5.1), which is essentially (5.12) of [B], it is confirmed that the difference between the values of $\overline{N}$ computed by either formula is exceedingly small, reaching a maximum of 1.6 mm for the poles. Although the refined formula giving $\overline{N}$ (i.e., A5.26 above) has proved to be little different from the approximate formula (5.12) of [B], it is useful in more respects than one: it confirms the development in both [B] and in this study to a certain extent since slightly different routes have been taken in its derivation; it offers a better insight into the effect of the moon's inclination on the average equilibrium tidal undulation; and it offers the means to compute the average equilibrium tidal undulation to a high accuracy (1 mm) if such accuracy is needed.

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